ABSTRACT

A boundary integral equation technique, involving surface vorticity distributions as the boundary unknowns, is applied to solve the unsteady marine propeller performance problem, resulting from the operation of a propeller in inclined flow or in the ship's wake, under the assumption of weak interaction between the sheared onset flow and the propeller induced velocity. The mathematical formulation is based on a system of second-kind, Fredholm-type boundary integral equations, obtained by the velocity representation theorem. This system is characterized by weakly singular kernels, which permits an efficient and accurate inversion. A pressure-type Kutta condition is satisfied along the leading edge of the blades. Extensive comparisons are presented between the present method, other boundary element techniques and experimental measurements, showing that reliable information concerning the time-history of the blade pressure distribution and the integrated propeller characteristics can be obtained, supporting, thus, the detailed design procedure of marine propellers. Our numerical results are focused on the unsteady performance of propellers with non-conventional blade geometry (increased skew, pitch and expanded area ratio, with large chord-length distribution), operating in inclined flow conditions, and in circumferentially varying incoming flow, representing the ship's wake. Although the present work is restricted to potential flow applications, the surface vorticity formulation can be used to couple rotational flow and potential flow in a direct way. This approach, in conjunction with solvers of the vorticity transport equation, or the Euler equations, in three dimensions, permits us to improve the numerical predictions concerning the unsteady propeller analysis and the propeller-hull interaction problems, overcoming the limitations of weak interaction assumption.

INTRODUCTION

The full treatment of the coupled propeller-hull hydrodynamic interaction problem, in the framework of Navier-Stokes equations, is extremely demanding computationally. Recent CFD investigations of the viscous aspects of the hull-propeller interaction are based on Reynolds Averaged Navier Stokes (RANS) equations, in conjunction with various simplifications concerning the action of the propeller, as, e.g., actuator disk models, Tzabiras (1996, 1997), or lifting surface models, Stern et al (1994), to calculate the body force term.

On the other hand, the detailed propeller analysis is still based on potential flow approximations, ITTC (1999). On this basis, the solution of the unsteady propeller performance problem, operating in a non-uniform inflow representing the ship's wake, is still very useful for engineering applications, offering valuable information concerning the propeller time-dependent integrated and distributed characteristics that are important for large class of further studies, as concerns propulsive system performance, intermittent cavitation, hull vibration etc. Moreover, it provides us with the propeller induced velocity field and the body forces to be used by three-dimensional solvers of the Euler equations in order to obtain improved prediction of the non-axisymmetric effective wake, Choi and Kinnas (2001).

In the present work, the unsteady propeller analysis problem is treated the framework of potential, lifting flow applications, under the assumption of weak interaction between the propeller and the sheared onset flow. To this aim, the most popular ways to reduce problems involving the Laplace equation to equivalent boundary integral equations are the method of potentials and the direct method, which is based on Green's representation formulae, Kinnas (1996). This reduction is by no means unique, therefore, a constantly growing number of equivalent formulations is disposed today for the solution of the problem under consideration, Katz and Plotkin (1991). In a similar way, it is possible to reduce the Neumann boundary value problems formulated with respect to the velocity field, to boundary integral equations. This can be succeeded, also in a direct way, by applying the Poincaré representation formula for three dimensional vector fields, Brard (1972), Belibassakis and Politis (1995). Accordingly, a Fredholm-type, second-kind system of boundary integral equations is derived, the unknown of which is the tangential component of the vector field on the boundary, rotated by 90 degrees. Because of the close relationship of this surface singularity distribution with the notion of vortex sheets, it is termed herein the surface vorticity distribution. The system of surface vorticity boundary integral equations is characterised by weakly singular kernels, permitting efficient integration and accurate inversion of the system. It is worth noticing here that a similar representation has been recently proposed for the representation of free-surface flows and ship waves by Noblesse (2001).

Applying the surface vorticity formulation to the propeller-induced velocity field an equivalent system of second-kind Fredholm-type boundary integral equations is derived, with time-dependent unknown the vorticity components on the solid boundary. The latter are calculated by the time-dependent boundary condition. Comparisons between the present method, other established boundary element techniques and experimental measurements are presented, showing that reliable information concerning the time-history of the pressure distributions and integrated propeller blade characteristics can be obtained, supporting the detailed design procedure of marine propellers.
THE SURFACE VORTICITY B.I.E. METHOD

Let $\Omega$ be a bounded, open, simply connected domain in $\mathbb{R}^3$, with a boundary $\partial \Omega$, and $Q = \mathbb{R}^3 \setminus \Omega$ denote the complementary exterior domain. The representation theorem for a continuously differentiable vector field $\mathbf{u}$, Brand (1972), states that the vector function can be reproduced, if its divergence $\nabla \mathbf{u}$ and rotation $\mathbf{V} \times \mathbf{u}$, at every point in the domain(s) are known, and if its boundary values are properly specified. The theorem, presented in a unified formula for both the exterior and the interior vector fields, reads:

$$
\mathbf{u}(P) = \int_{\partial \Omega} \left( \mathbf{n} \cdot \mathbf{V} \times \mathbf{u} \right) \times \hat{\mathbf{G}}(Q, P) d\mathbf{S}(Q) + \int_{\partial \Omega} \mathbf{V} \cdot \mathbf{G}(Q, P) d\mathbf{V}(Q)
$$

where $\mathbf{n}$ is the positively oriented, unit normal vector to the boundary $\partial \Omega$ with respect to the exterior domain, $\mathbf{V} \times \mathbf{u}$ denotes the discontinuity of the vector field across the boundary, and $\hat{\mathbf{G}}(Q, P)$ is the gradient of the fundamental solution of the Laplace equation in $\mathbb{R}^3$:

$$
\hat{\mathbf{G}}(P, Q) = \frac{\hat{\mathbf{x}}(Q) - \hat{\mathbf{x}}(P)}{4\pi |\hat{\mathbf{x}}(Q) - \hat{\mathbf{x}}(P)|}.
$$

It is assumed that the boundary $\partial \Omega$ can be locally represented through a regular parametric representation of the form: $\hat{\mathbf{x}} = \hat{\mathbf{x}}(\xi^1, \xi^2)$, mapping an open parametric domain $\{\xi^1, \xi^2\} \subset \mathbb{E} \subset \mathbb{R}^2$ bijectively onto a surface patch of $\partial \Omega$.

Then, the vectors $\hat{\mathbf{e}}_a = \frac{\partial \hat{\mathbf{x}}}{\partial \xi^a}$, $a = 1, 2$, form a covariant local base on the boundary surface. The direction of the parametric curves is properly selected, in order to produce the outward unit normal vector on the boundary and the surface metric as follows

$$
\mathbf{n} = \hat{\mathbf{e}}_1 \times \hat{\mathbf{e}}_2, \quad a = |\hat{\mathbf{e}}_1 \times \hat{\mathbf{e}}_2|.
$$

The surface distribution $\hat{\mathbf{y}} = \mathbf{n} \times \mathbf{V} \times \mathbf{u}$, appearing in the second term of the right hand side of equation (1), is termed the surface vorticity distribution, taking into account its similarity with the notion of vortex sheets. The surface distribution $\sigma = \mathbf{n} \times \mathbf{V} \times \mathbf{u}$ is the surface source distribution. Resolved into its contravariant components in the surface curvilinear coordinate system the surface vorticity distribution is also written in the form

$$
\hat{\mathbf{y}} = \mathbf{n} \times \mathbf{V} \times \mathbf{u} = \gamma^1 \hat{\mathbf{e}}_1 + \gamma^2 \hat{\mathbf{e}}_2,
$$

where $\gamma^a$, $a = 1, 2$, are its contravariant components.

Dealing with Neumann-type, exterior, incompressible flow ($\nabla \mathbf{u} = 0$) problems, a direct way to reduce into boundary integral equation is possible, Belibassakis & Politis (1995). As a final result, the following vectorial integral equation, involving the surface vorticity $\hat{\mathbf{y}}(P)$ on the boundary $\partial \Omega$ is obtained.

For $P, Q \in \partial \Omega$,

$$
\frac{\hat{\mathbf{y}}(P)}{2} - \hat{\mathbf{n}}(P) \times \int_{\partial \Omega} \hat{\mathbf{y}}(Q) \times \hat{\mathbf{G}}(Q, P) d\mathbf{S}(Q) =
$$

$$
= \hat{\mathbf{n}}(P) \times \left\{ \int_{\partial \Omega} \mathbf{V} \times \hat{\mathbf{G}}(Q, P) d\mathbf{V}(Q) \right\}.
$$

The above representation permits us to directly couple a rotational flow, for which a velocity potential cannot be defined, and a potential flow, without having to solve an integral equation in order to match the velocity in the interface between the vortical and the potential surges. More details about the above surface vorticity boundary integral formulation can be found in Belibassakis and Politis (1995, 1998), where also numerical results are presented concerning its application to three-dimensional potential lifting flows.

THE UNSTEADY PROPELLER ANALYSIS PROBLEM

We consider a set of identical, symmetrically arranged blades mounted on a hub, rotating at a constant angular velocity $\omega$ about the axis of symmetry and translating at a constant velocity $\mathbf{U}$ in unbounded, incompressible, inviscid fluid. Without the loss of generality, a right-handed propeller is assumed. $S_h$ denotes the solid surface of each symmetrical part. The solid boundary is then composed by the union of its parts: $\partial \Omega = \cup S_h$. We also consider the following Cartesian coordinate systems, (i) the motionless, inertial coordinate system, denoted by $\{z_i\}$, (ii) the moving, inertial coordinate system, attached to the ship, and denoted by $\{y_i\}$, which is translated steadily with respect to the former, and (iii) the propeller-fixed coordinate system, denoted by $\{x_i\}$, which is rotating at a constant angular velocity $\omega$ and, at the same time, it is translating at a constant velocity $\mathbf{U}$ with respect to the motionless system $\{z_i\}$. The cylindrical coordinate system corresponding to $\{x_i\}$ is denoted by

$$
\{x, r, \theta\} = \left\{x = x_i, r = \sqrt{(x_i)^2 + (x_j)^2}, \theta = \tan^{-1}(x_i/x_j)\right\}.
$$

The total velocity field in the propeller fixed frame of reference consists of the following components:

$$
\mathbf{w} = \left[ -\omega \times \hat{x} - \mathbf{U} + \hat{A}(\hat{x}, t) + \hat{u}(\hat{x}, t) \right] = \hat{q}(\hat{x}) + \hat{A}(\hat{x}, t) + \hat{u}(\hat{x}, t),
$$

where $-\omega \times \hat{x}$ and $-\mathbf{U}$ are the relative velocities due to rotation and translation respectively, $\hat{A}(\hat{x}, t)$ represents the ship’s effective wake distribution and $\hat{u}(\hat{x}, t)$ the propeller-disturbance velocity field (the propeller induced velocity). The effective wake, is considered to be decomposed as follows,

$$
\hat{A}(\hat{x}, t) = \hat{A}^{u}(\hat{x}, t) + \hat{A}^{\omega}(\hat{x}, t).
$$
where \( \tilde{A}^{e}(\tilde{x};t) \) denotes the nominal wake distribution (existing in the absence of the propeller interference) and \( \tilde{A}^{d}(\tilde{x};t) \) denotes the rotational velocity field associated with the redistribution of vorticity. For an observer in the rotating frame of reference the first terms in the right hand side of equation (6), considered together and denoted by \( \tilde{q}(\tilde{x})+\tilde{A}(\tilde{x};t) \), constitute the propeller onset flow, which is also termed the undisturbed velocity field.

Weak interaction between the sheared onset flow and the propeller generated flow basically states that the viscous wake \( \tilde{A}(\tilde{x};t) \) interferes with the formation of the propeller induced velocity field \( \tilde{u}(\tilde{x};t) \), but the converse effect can be neglected. The nominal wake is a real, self-existent flow, which has by itself to satisfy the equations of fluid motion. This incompressible component is considered time-invariant in the ship-fixed frame of reference \( \{\gamma_r\} \). By considering the blade tip speed \( U_f = \sqrt{\omega R^2 + U_t^2} \), where \( \omega = \omega_0 \), \( U_t = [U] \) and \( R \) is the propeller tip radius, as the reference value for magnitude estimates concerning vorticity, and the propeller rotational speed \( \omega \) as the reference value for magnitude estimates concerning vorticity, the following assumptions are made for the nominal wake,

\[
\frac{\tilde{A}^{(e)}}{U_f} = O(\varepsilon), \quad \frac{\nabla \times \tilde{A}^{(e)}}{\omega} = O(\varepsilon), \quad (8)
\]

where \( \varepsilon \) is a small, non-dimensional parameter. Furthermore, the shear interaction term \( \tilde{A}^{d}(\tilde{x};t) \), considered in the propeller frame of reference, as well as its time and space derivatives are assumed to be of second order. The propeller induced velocity field \( \tilde{u}(\tilde{x};t) \) is assumed to be of the same order as the nominal wake, i.e. \( \tilde{u}(\tilde{x};t)/U_f = O(\varepsilon) \). By noting that the absolute velocity of the fluid particles \( \tilde{A}(\tilde{x};t)+\tilde{u}(\tilde{x};t) \) is a purely incompressible field, we obtain that the propeller induced velocity field should also satisfy by itself the equation of continuity. In addition, this component characterizing the disturbance velocity due to propeller's operation, should diminish far from the body.

The total velocity field \( \tilde{u}(\tilde{x};t) \) has to satisfy the Euler equations. Also, the nominal ship's wake is a real, self-existent flow. Therefore, this flow considered in the propeller-fixed frame of reference, must also satisfy the Euler equations. By subtracting (term by term) these equations, we finally obtain,

\[
\frac{\partial \tilde{u}}{\partial t} + (\nabla \times \tilde{u}) \times \tilde{w} = \nabla \times \tilde{A}^{(e)}(\tilde{x};t) + \left( \frac{p - p^{(i)}}{\rho} \right) + \left[ \frac{\tilde{q}^2}{2} - \tilde{A}^{(d)}(\tilde{x};t)^2 \right] + \text{higher-order terms}, \quad (9)
\]

where \( p^{(i)}(\tilde{x};t) \) stands for the onset flow pressure distribution.

Furthermore, the vorticity associated with the propeller induced velocity field \( \tilde{u}(\tilde{x};t) \) is considered concentrated on two dimensional surfaces \( \{S_{w_i}\}_{i=1,N} \) emanating from the trailing edge of each blade. This can be formally expressed by the following relations

\[
\nabla \times \tilde{u} = \delta_2 \cdot \tilde{\gamma}, \quad \text{in } \Omega_s, \quad \text{and} \quad \tilde{\gamma} = \tilde{n} \times <\tilde{u}>, \quad \text{on } S_{w}, \quad (10)
\]

where \( \Omega_s \) is the exterior to the solid body domain, \( \delta_2 \) is the Dirac-delta distribution uniformly spread over the trailing vortex sheets \( \{S_{w_i}\}_{i=1,N} \), and \( < > \) is used to denote the discontinuity of a quantity over the surface. A direct consequence of these relations as concerns the propeller disturbance velocity field is that it can be considered to be generated by a potential

\[
\tilde{u}(\tilde{x};t) = \nabla \cdot \Phi(\tilde{x};t), \quad \text{in } \Omega_s \setminus S_w, \quad (11a)
\]

where \( S_w = \cup S_{w_i} \). On the basis of above considerations the unsteady propeller performance problem reduces to the following boundary value problem with time-dependent boundary conditions:

\[
\nabla \times \left( \Phi(\tilde{x};t) \right) = 0, \quad \text{in } \Omega_s \setminus S_w, \quad (11a)
\]

\[
\frac{\partial \Phi}{\partial t} + \frac{p - p^{(i)}}{\rho} + \frac{1}{2} \left( \left| \tilde{q} \right|^2 - \tilde{A}^{(d)}(\tilde{x};t)^2 \right) = 0, \quad \text{in } \Omega_s \setminus S_w, \quad (11b)
\]

\[
\tilde{u} \cdot \tilde{n} = -\tilde{u} \cdot \left( \tilde{q}(\tilde{x}) + \tilde{A}^{(d)}(\tilde{x};t) \right), \quad \text{on } S_w, \quad (11c)
\]

\[
\left| \tilde{u}(\tilde{x};t) \right| < \infty, \quad \text{Kutta condition at trailing edge}, \quad (11d)
\]

\[
< p(\tilde{x};t) > = 0, \quad \frac{DF_{\delta_i}}{Dt} = 0, \quad \text{on } S_w, \quad (11e)
\]

In the above equations \( F_{\delta_i}(\tilde{x};t) \) are the equations of the trailing vortex surfaces \( S_{w_i} \) and \( D(\cdot)/Dt \) denotes the surface material derivative. The last two conditions \( (11e) \) simply express the fact that blade vortex sheets are represented by material force-free surfaces.

As it is usually the case with lifting flow applications, the Kutta-Joukowski hypothesis, requiring finite velocity at the trailing edge, is enforced indirectly. The generally accepted as optimum indirect formulation of the Kutta condition, Jessup (1989), Kimnas (1996), is the condition of equal limit pressures on the upper and lower surfaces, as the trailing edge is approached. This variant has also been used in the present work. It is more convenient to apply the equal pressure condition on the side of the body, although it can also be applied on the two sides of the wake surface in the immediate vicinity of the trailing edge.

A time marching integration method is utilized for the numerical solution of the boundary value problem with time dependent boundary conditions \( (11) \), assuming an initial state of rest. Since the flow has been assumed incompressible, the influence of the momentary boundary condition is immediately radiated across the whole fluid region. Therefore, the kinematics of the time-dependent propeller-analysis problem can be numerically treated by substituting the instantaneous boundary condition at each, discrete time step. However, the time discretization has to be fine enough, so that the time derivatives appearing in the formulation can be well approximated by a finite difference scheme. In accordance with the above considerations, one of the principal features of the unsteady propeller analysis problem is a memory effect. Information concerning the time history of the
fluid motion is stored in the disturbance potential $\Phi(\mathbf{x},t)$ and in the blade vortex wake, and influences the solution at subsequent time levels. It is also to be remarked here that the approximate version of Bernoulli’s equation (11b), obtained on the basis of weak interaction hypothesis, enables full decoupling between the calculation of disturbance velocity $\mathbf{u}(\mathbf{x},t)$ and the calculation of propeller-induced pressure $p - p_i$.

Considering the subdivision of each blade trailing vortex sheet, $S_{q_k} = S_{q_k}^{\text{init}} \cup S_{q_k}^{a}$, $k = 1, N$, where $S_{q_k}^{\text{init}}$ denotes the initial portion of $S_{q_k}$ adjacent to the trailing edge of the blade, and $S_{q_k}^{a}$ the remaining part, extended to infinity downstream, the boundary integral equation, governing the unsteady propeller performance problem is written (for $P, Q \in S_q$) as follows,

$$
\frac{1}{2} \sum_{k=1}^{N} \int_{S_{q_k}} \gamma^a(Q;t) \mathbf{e}_x(Q) \times \mathbf{G}(Q,P) dS(Q) - \mathbf{n}(P) \times \sum_{k=1}^{N} \int_{S_{q_k}} \gamma^a(Q;t) \mathbf{e}_x(Q) \times \mathbf{G}(Q,P) dS(Q) =
$$

$$
= \mathbf{n}(P) \times \sum_{k=1}^{N} \left\{ \int_{S_{q_k}} g(Q,t) \mathbf{G}(Q,P) dS(Q) + \int_{S_{q_k}} \gamma^a(Q;t) \mathbf{e}_x(Q) \times \mathbf{G}(Q,P) dS(Q) \right\},
$$

where,

$$
g(Q,t) = -\mathbf{n}(Q) \cdot \mathbf{q}(Q,t) + \mathbf{A}(Q,t), \quad Q \in S_q,
$$

are the boundary conditions.

The pressure type Kutta condition is applied to determine the intensity of the trailing vorticity $\{\gamma^a_{\alpha}\}$, $\alpha = 1, 2$, on $S_{q_k}^{a}$. Using the Bernoulli equation (11b), the Kutta condition is written as:

$$
\frac{\partial (\Phi_{\alpha} - \Phi_{\beta})}{\partial \mathbf{x}} + \frac{1}{2} \left\{ \mathbf{w}(\mathbf{x}) \cdot \left[ \mathbf{w}(\mathbf{x}) \right] - \mathbf{w}(\mathbf{x}) \right\} = 0,
$$

where $(\alpha)$ and $(\beta)$ indicate corresponding points on the suction and pressure surface, at the trailing edge of the blades, respectively, and $\mathbf{w}$ is the total fluid velocity on the solid boundary, expressed directly in terms of the surface vorticity as

$$
\mathbf{w}(P,t) = \mathbf{q}(P,t) + \mathbf{A}(P,t) + g(P,t) \cdot \mathbf{n}(P) + \mathbf{\gamma}(P,t) \times \mathbf{n}(P), \quad P \in S_q.
$$

The disturbance potential $\Phi(P,t)$, for $P \in S_q$ is conveniently calculated by integrating along the boundary surface parametric curves:

$$
\Phi(P,t) = \int_{\mathbf{c}} \sqrt{a\left(\xi^2,\xi^2\right)} \gamma^a\left(\xi^2,\xi^2;t\right) d\xi^2,
$$

For the numerical solution of the integral equation (12), the propeller is discretised into quadrilateral panels, ensuring continuity of the geometry. The surface vorticity components are approximated by low-degree polynomials in each panel. In must be stressed however that the surface vorticity is directly proportional to the corresponding velocity, see, e.g., Eq. (4), and thus, a higher degree of approximation is achieved as concerns the associated potential. More details about the numerical implementation and solution of the surface vorticity boundary integral method, applied both to the steady and to unsteady propeller analysis problems, can be found in Belibassakis and Politis (1995, 1998).

**Numerical Results**

In order to better illustrate the special features of the present vorticity-based unsteady boundary integral equation method, in this section, numerical results are presented and compared with available experimental measurements, as well as, with numerical predictions obtained by other methods based on potential formulation. We concentrate in this section on results concerning the unsteady performance of propellers with non-conventional blade geometry, operating in inclined flow conditions, and in circumferentially varying incoming flow, representing the ship’s wake. Corresponding results obtained by the present method concerning the unsteady propeller performance of more classical blade geometries have been presented and discussed in Belibassakis and Politis (1998).

**Modeling of the propeller blades trailing vortex sheets**

To account for the effects of trailing vorticity, a free vortex sheet downstream the lifting sections of the blades incorporated to the present model. Since the geometry of this additional boundary is unknown, it has to be determined as part of the solution. However, if the shape of the idealized hydrodynamic wake (the wake model) can be estimated in advance, the problem is reduced to an integral equation over the solid surface coupled with the Kutta condition. Wake models are usually constructed exploiting information from experimental observations and measurements of the relative flow quantities downstream the propeller, and by making systematical use of the non-linear numerical tools in the case under consideration.

Following previous investigators, Hoshino (1993), the initial part of the propeller blade vortex sheets is mathematically represented by smooth surfaces constituting the blade-wake transition region. The ultimate wake region is either a generalized helicoidal surface or it degenerates into two sets of rolled-up hub and tip vortices per blade, representing experimental observations concerning the deformation of the propeller blade vortex sheets, at increased blade loading conditions. The radial distribution of the hydrodynamic pitch angle is selected to be given in terms of the main geometrical and hydrodynamic parameters. Within the transition wake region, the trailing vortex lines which emanate from the trailing edge of each blade are aligned to the local trailing edge angle bisector. Subsequently, moving in the downstream direction, they gradually deform in order to match smoothly the ultimate wake trailing vortex lines. In the case of the ‘deformed’ wake model, an additional contraction is superimposed.
NSRDC 4119, 3-bladed propeller in uniform flow.

Numerical predictions concerning the open water characteristics of this propeller have been obtained by subdividing each blade into 11 spanwise by 50 chordwise elements. In Figure 1 the results of our calculation are compared with Hoshino’s panel method (1993) and experimental data. The comparison concerns the integrated open-water propeller characteristics. Both numerical methods are in good agreement with experimental measurements in an extended range of advance coefficients.

In Figure 2 the calculated blade sectional pressure distribution for the propeller operating at  $J=0.833$ (design point) and at $r/R=0.70$ is compared with similar calculations by Hoshino (1993) and the experimental data (Jessup, 1989). Small discrepancies between the two panel methods appearing close to the trailing edge of the blades are mainly due to the different characteristics of the blade wake model used by the present method and by Hoshino’s panel method.

NSRDC 4679, 3-bladed propeller in inclined flow.

This propeller is characterised by increased skew, pitch and expanded area ratio, producing a much larger chord length distribution, as compared to the blade geometry of a conventional propeller. The flow conditions about the tip of this propeller exhibit similarity to the flow at the tip of a delta wing, therefore, the pressure distribution at the outer radii is also expected to be dominated by three-dimensional phenomena, such as significant cross-flow and tip vortex roll-up. This propeller was tested (Jessup, 1982) in uniform flow, driven at a shaft inclination of 7.5 degrees with respect to the horizontal. Numerical predictions obtained by the present method for the propeller operating at the design point, $J=1.078$, and at more increased blade loading, $J=0.719$, are presented in Figures 3 to 10.

Due to the large amount of calculations (in the time domain) a coarser grid (10 spanwise by 30 chordwise elements per blade) has been used for numerical simulation. Detailed results concerning the unsteady blade pressure distribution, for the propeller operating at $J=1.078$ (design) and $J=0.719$ (increased blade loading) are presented comparatively to the measured data (Jessup, 1982) in Figures 3 to 6, and in Figures 7 to 12, respectively.

More specifically, the mean pressure coefficient for the propeller operating at the design point and at the blade sections $r/R=0.70$ and $r/R=0.90$ is shown in Figures 3 and 4, respectively. The corresponding first harmonic amplitudes of the pressure coefficient are shown in Figures 5 and 6, respectively. The medium pressure coefficient for the propeller operating at increased loading ($J=0.719$) and at the same as before blade sections are shown in Figures 7 and 9. The corresponding first harmonic amplitudes of the pressure coefficient are shown in Figures 8 and 10.

From the above figures we can observe that the present method results concerning the mean and the first harmonic amplitude of the blade pressure are in a good agreement with experimental data. The comparison concerning the first harmonic reveals small discrepancies, especially in the face (pressure) side and away from the leading edge. However, since the corresponding amplitudes are significantly smaller than the average pressure, this discrepancy does not affect the instantaneous blade loads.

HSP ‘Seiun-Maru’, 5-bladed propeller in unsteady flow.

This Highly Skewed Propeller was tested in unsteady flow conditions, operating in a circumferentially varying (ship’s) wake, containing axial, tangential and radial components. In the present method the ship’s wake is represented by means of a Fourier series. Only the first 9 harmonics have been retained in the calculations by the present method.

A computational grid consisting of 10 spanwise by 30 chordwise elements per blade has been used for numerical simulation. Detailed results concerning the integrated blade characteristics and the unsteady blade pressure distribution, for the propeller operating at $J=0.851$ are presented comparatively to the calculations by Hoshino (1993) and to measured data (Ukon et al, 1991) in Figures 11 to 15.

In Figure 11 the time history of the blade thrust and torque coefficients in one propeller revolution is presented, as obtained by the present method and as calculated by Hoshino (1993). The numerical results obtained by the two panel methods are in good agreement.

The instantaneous pressure coefficient at the section $r/R=0.70$, where the blade load is maximized, and at the blade angular positions 0 and 90 degrees is presented in Figures 12 and 13, respectively. The numerical predictions by the two panel methods are again found in good agreement. Again, small discrepancies, due to different blade-wake and ship-wake representation used by the two models, appear near the trailing edge of the blades, and are more profound at the top position of the blade, where the instantaneous loading of the blade is maximum.

The histories of the pressure coefficient in one propeller revolution are presented in Figures 14 and 15. More specifically, the blade pressure coefficient at the blade section $r/R=0.70$, at the chordwise position $x/c=0.25$, is presented in Figure 14, and at $x/c=0.60$ in Figure 15. Results on both the blade face and back sides are presented.

Differences between the results of the two panel methods observed near the trailing edge and at the top position of the blade, are due to the different blade-wake and ship-wake representations used by the two panel methods.

![Open Water Characteristics of DTMB P4119](image-url)

**Figure 1:** Propeller P4119 in steady flow
**Figure 8:** Propeller P4679 in inclined flow

**Figure 9:** Propeller P4679 in inclined flow

**Figure 10:** Propeller P4679 in inclined flow

**Figure 11:** HSP propeller in unsteady flow

**Figure 12:** HSP propeller in unsteady flow

**Figure 13:** HSP propeller in unsteady flow
CONCLUSIONS

The unsteady propeller analysis problem is treated in the framework of potential flow applications, under the assumption of weak interaction between the propeller and the sheared onset flows. A boundary integral equation technique, involving the surface vorticity distribution as the unknowns, has been developed and applied to the solution of the Neumann boundary value problem of three dimensional velocity fields. Subsequently, this method has been applied to the unsteady propeller performance problem, formulated as a lifting flow problem, with respect to the propeller-induced velocity field.

Comparisons are presented between the present method, other boundary element methods and measured data. Our numerical results are focused on the unsteady performance of propellers with non-conventional blade geometry (increased skew, pitch and expanded area ratio, with large chord-length distribution), operating in inclined flow conditions, and in circumferentially varying incoming flow, representing the ship's wake. It is illustrated that reliable information concerning both the blade pressure distributions and the unsteady integrated propeller characteristics can be obtained, supporting the detailed design procedure of marine propellers. Although the present application is restricted to irrotational flow, the surface vorticity method can be used, in conjunction with solvers of the vorticity transport equation for inviscid flows, or solvers of the Euler equations in $R^3$, in order to overcome the limitations of the weak interaction assumption and improve the numerical predictions concerning the unsteady propeller analysis problem and the determination of the effective wake.

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Figure 14: HSP Propeller in unsteady flow

Figure 15: HSP Propeller in unsteady flow