EXTREME VALUE PREDICTION ON DECREASING DEPTH BY MEANS OF A NONLINEAR WAVE TRANSFORMATION MODEL

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ABSTRACT

In the present work a weakly nonlinear wave model originally developed by Rebaudengo Landò et al (1996) is applied to the transformation of wave spectra from offshore to nearshore, and subsequently, it has been systematically applied to the derivation of long-term time series of spectral wave parameters on decreasing depth from corresponding offshore wave data. The derived long-term series of nearshore parameters have been used as input to a new method, recently developed by Stefanakos & Athanassoulis (2006), for calculating return periods of various level values from nonstationary time series data. The latter method is based on a new definition of the return period, that uses the MEan Number of Upcrossings of the level \( x^* \) (MENU method), and it has been shown to lead to predictions that are more realistic than traditional methods. To examine the effects of bottom topography on the nearshore extreme value predictions, Roseau (1976) bottom profiles have been used for which analytical expressions are available concerning the reflection and transmission coefficients. A parametric (JONSWAP) model is used to synthesize offshore spectra from integrated parameters, which are then linearly transformed based on the previous transmission coefficient to derive first-order nearshore wave spectra. Second-order random sea states have been simulated by following the approach of Hudspeth & Chen (1979) (see also Langley 1987, Lando et al 1996), exploiting the quadratic transfer functions on decreasing depth to calculate the second-order nearshore spectra. Finally, wave parameters are extracted from the nearshore spectra by calculating the first few moments.

1. INTRODUCTION

Traditionally, when very large datasets of wave data are available, return periods of significant wave height are calculated by means of the annual maxima, a widely used method known as the Gumbel's approach (Gumbel 1958). However, such large datasets are rare, and, thus, various alternative methods have been proposed. Among them, the major successor is the so-called Peaks-Over-Threshold (POT) method, introduced in ocean engineering the last decade (van Vledder & Zitman 1992, van Vledder et al 1994, Ferreira et al 1998, Naess 1998). Although POT method uses more data for the extreme-value predictions, i.e. the values above a relatively high threshold, it does not fully exploit the stochastic structure of the data.

A further step would be to model a time series as a stochastic process and calculate its extremes based on the available theory (Rice 1944/45, Leadbetter et al 1983). Recently, various enhanced models for the representation of time series of wind and wave parameters have been proposed; see, e.g., Athanassoulis & Stefanakos (1995), Soares et al (1996), Cunha et al (1999), Stefanakos et al (2001), Stefanakos et al (2003), Stefanakos et al (2005). In the above models, apart from the stochastic character of the series, the dependence structure and the seasonality exhibited are appropriately modelled also. Then, the theory of periodically correlated stochastic processes can be used, permitting us to significantly improve extreme-value predictions.

In the present work, a new method for calculating return periods of various level values from long-term nonstationary time series data is applied to both offshore (input) and nearshore (derived) time series of wave parameters. Reliable, long-term offshore time series of wave parameters are available from various sources, such as satellites, meteorological wave models and buoys or platforms, some of them with extended (global) coverage; see, e.g. Barstow et al (2003). In order to derive nearshore wave data from existing offshore ones, one approach is to employ deterministic wave models to derive the transformation of incident wave spectra over variable bathymetry regions. One possible candidate model for this job is the weakly nonlinear spectral model developed by Rebaudengo Landò et al (1996), for uni-directional and multi-directional random wave propagation on decreasing water depth, taking into account shoaling, refraction
and reflection phenomena, as well as saturation and second-order interaction processes. The latter is considered to be important, especially in the case of severe offshore sea states.

Following this approach, for each wave condition in the offshore time-series of data we consider irregular wave propagation in a shoaling region, which can be interpreted as an offshore to nearshore wave transformation of the time series. For each point in the offshore series, an incident wave spectrum \( S^{(i)}(f) \), where \( f \) denotes the frequency, realised by means of the JONSWAP parametric model, is associated with the random wave characteristics in the far up-wave region \( D^{(0)} \); see Fig.1. The latter is considered as the offshore excitation of our hydrodynamical system.

The procedure applied to derive nearshore wave parameters from offshore ones consists of the following main steps. First, the linear monochromatic wave transformation problem is solved in the frequency domain, for many discrete frequencies densely covering the band of frequencies involved in the offshore spectra. One possible candidate model for this job is the coupled-mode model developed by Athanassoulis & Belibassakis (1999) for wave propagation over the variable bathymetry region \( D^{(2)} \). This model has been further extended to 3D by Belibassakis et al. (2001), and various applications of it to the offshore to nearshore wave spectrum transformation can be found in Athanassoulis et al. (2003) and Gerostathis et al. (2005). In the present work, in order to examine the effects of bottom topography on the nearshore extreme value predictions, without contamination by numerical errors (especially in the high-frequency band), we use Roseau (1976) shoaling bottom profiles, for which analytical expressions are available concerning the reflection and transmission coefficients. Then, the nearshore spectra in the shallow water region \( D^{(3)} \) (see Fig.1) are obtained on the basis of responses of linear distributed systems. The derived nearshore spectra are treated as saturation processes, see Le Méhauté & Wang (1992). At a next stage, second-order random sea states have been simulated in the shallow water region \( D^{(3)} \) by following the approach of Hudspeth & Chen (1979), by exploiting the quadratic transfer functions on decreasing depth to calculate the second-order nearshore spectra; see also Langley (1987), Lando et al (1996). Finally, wave parameters are extracted from the derived nearshore spectra by calculating the first few moments, and long-term time-series of nearshore data are composed.

The derived nearshore data have been used as input to a new method, recently developed by Stefanakos & Athanassoulis (2006), for calculating return periods of various level values from nonstationary time series data. The method is based on the MEAN Number of Upcrossings of the level \( x* \) (MENU method) and has been recently developed by Stefanakos and Athanassoulis (2006); see also Stefanakos and Monbet (2006). A similar approach based on the characteristic function of the Gaussian process is shown in Naess (2001). The present method results are compared with corresponding ones obtained by traditional methods (Gumbel's approach and POT method) and they are found to be in good agreement.

### 2. TRANSFORMATION OF WAVE SYSTEMS

For simplicity, in this work we restrict ourselves to two dimensions, i.e. horizontal wave propagation (along the \( x \)-axis) is considered; see Fig.1. The incident spectrum corresponding to each offshore wave condition (exciting the present hydrodynamical system) is modelled by a parametric JONSWAP model, used to synthesize offshore spectra from integrated wave parameters. The form of the unidirectional JONSWAP spectrum considered in this work is given by

\[
S^{(i)}(f) = \frac{a_0 g^2}{(2\pi)^3 f^5} \exp \left[ 1.25 \left( \frac{f}{f_p} \right)^4 \right] \times \\
\exp \left[ \ln \gamma_0 \exp \left[ -0.5 \left( \frac{f}{f_p} - 1 \right)^2 \right] \frac{1}{\sigma_0} \right], \tag{2.1}
\]

where \( a_0 \) is the equilibrium parameter, \( g \) is the acceleration of gravity, \( \gamma_0 \) the enhancement parameter, \( \sigma_0 \) the peak width parameter and \( f_p \) is the peak frequency.

On the basis of the theory of responses of distributed linear systems, first-order nearshore wave spectra are then obtained by linearly transforming the incident spectrum, using the frequency-dependent transmission coefficient, as follows

\[
S^{(3)}_T(f) = S^{(i)}(f) K_T^2(f), \tag{2.2}
\]

where \( K_T(f) \) denotes the transmission coefficient of waves propagating over the variable bathymetry region.

For general bottom profiles, \( K_T(f) \) along with all other important quantities associated with the wave field can be very efficiently obtained by applying the coupled-mode theory (Athanassoulis & Belibassakis 1999) in 2D or by its 3D extension (Belibassakis et al. 2001). However, in the present work transmission and reflection coefficients are analytically obtained by employing Roseau (1976) profiles to model the shoaling region, as it will be described in more detail in the next section.

The representation of the power spectrum on decreasing depth can be enhanced if we take into account saturation processes in a consistent manner, as for example the TMAR model proposed by Gentile et al. (1994):

\[
S^{(3)}_{TMAR}(f) = \frac{a_0 g^2 \bar{U}_{0}^{2/3}}{(2\pi)^3 f^5} \exp \left[ 1.25 \left( \frac{f \sqrt{\chi}}{f_p \sqrt{\chi_p}} \right)^4 \right] \times \\
\exp \left[ \ln \gamma_0 \exp \left[ -0.5 \left( \frac{f \sqrt{\chi}}{f_p \sqrt{\chi_p}} - 1 \right)^2 \right] \frac{1}{\sigma_0} \right] \times \\
\left\{ \chi^{-2} \left[ 1 + 2 \sigma^2 \frac{\chi}{\sinh(2\sigma\sqrt{\chi})} \right] \right\}^{-1}, \tag{2.3a}
\]
where $a = 0.033$, $U_0 = f_s U_b / g$, $U_b$ is the wind speed at 10m height above the mean water surface, and the Kitaigorodskii’s depth functions are defined for each frequency in the shallow water depth $h$ as

$$\sigma_x(f, h) = 2\pi f \sqrt{h / g} ,$$  \hspace{1cm} (2.3b)$$
and

$$\chi(f, h) = \tanh^{-1}(\sigma_x^2 \chi),$$  \hspace{1cm} (2.3c)$$
see also Landò et al (1996). Noting that $\chi(f, h)$ is obtained for each frequency as the root of the above linear dispersion relation, the form (2.3a) represents a limit spectral form that cannot be exceeded.

Thus, the final form of the first-order spectrum describing the sea state in the shallow water region $D^{(3)}$ is

$$S_{L^{(3)}}^{(3)}(f) = \min_{f'} \left\{ S_{T}^{(3)}(f), S_{\text{multi}}^{(3)}(f, h) \right\} .$$ (2.4)

Next, we consider second order (weakly nonlinear) effects concerning the interaction between the wave components in the shallow water spectrum that is derived from the offshore one as a result of shoaling, refraction and saturation processes. To this respect, second-order random sea states are simulated in the shallow water region $D^{(3)}$ by exploiting the quadratic transfer functions on decreasing depth to calculate the second-order nearshore spectra, Hudspeth & Chen (1979), Laing et al (1986), by a procedure utilizing the first order spectrum relevant to the same depth.

Based on the above, the complete (up to second order) spectrum in the shallow water region $D^{(3)}$ is finally obtained

$$S_{NL}^{(3)}(f) = S_{L^{(3)}}^{(3)}(f) + \delta S^{(3)}(f) .$$ (2.9)

From the above final spectral form, its first few moments are easily obtained by frequency integration

$$m_{n}^{(3)} = \int_{f_{0}}^{f_{\infty}} f^n S_{NL}^{(3)}(f) df , \hspace{0.5cm} n = 0, 1, 2, \ldots ,$$ (2.10)

which permit the calculation of basic nearshore wave parameters, like the significant wave height $H_s = 4m_{1/2}$, etc.

### 3. WAVE TRANSFORMATION OVER SMOOTH SHOAL

In absence of dissipation and given proper lateral boundary conditions the flow in the shoaling over a bottom slope is irrotational and can thus be obtained by a numerical solution of Laplace's equation with bottom, surface, and lateral boundary conditions.
For waves of small amplitude, one possible candidate model for calculating linear wave propagation in variable bathymetry regions is the consistent coupled-mode model developed by Athanassoulis & Belibassakis (1999). As presented in Athanassoulis et al (2003), the above model supports the consistent description of the stochastic characteristics of all important physical quantities associated with the wave field (induced wave kinematics), taking fully into account refraction and diffraction effects, without restrictions concerning the mildness of the bottom slope. Extended comparisons presented in Athanassoulis et al (2003) revealed that the coupled-mode model provides enhanced predictions vs. simpler mild-slope type models, especially as concerns quantities defined near the bed surface, such as bottom velocities and pressures.

In the present work, numerical calculations are given for monotonically shoaling bottom profiles studied by Roseau (1976), for which the reflection and transmission coefficient are known analytically; see also Porter & Porter (2006). In this case, the bottom topography \( h(x) \) smoothly varies \( h'(x) < 0 \) only in \( a < x < b \), and is constant \( h(x) = h_i \), for \( x < a \), and \( h(x) = h_i \), for \( x > b \). The bottom surface defined here in the vertical \((x-z)\) strip is given by the real and imaginary part of the complex parametric function of the complex variable \( Z(\xi) \),

\[
Z(\xi) = x + iy = h_i(\xi - ia) + \frac{h_i - h_0}{\alpha} \ln(\exp(\xi - ia) + 1),
\]

where \( \alpha \) is an angle parameter, \( 0 < \alpha < \pi / 2 \), essentially controlling the length of the transition region. For example, in Fig.2 we present an example of the Roseau profile for \( h_i = 30 \text{m}, h_0 = 4 \text{m}, \) and \( \alpha = 15\pi / 180 \). In this case the essential length of the intermediate region \( D^{(2)} \) is of the order of 600m. Also, in the same figure the calculated values of the wave potential on the free surface, which is proportional to the free-surface elevation, are plotted over the shoal, for a frequency \( f = 0.127\text{Hz} \) (corresponding to angular frequency \( \omega = 0.8\text{rad/s} \)).

In the case of a monochromatic wave train of frequency \( f \) incident to the above profile, the reflection coefficient is analytically calculated as follows

\[
K_r(f) = \left[ \frac{\tanh \pi \lambda - \tanh \pi \mu}{\tanh \pi \lambda + \tanh \pi \mu} \right],
\]

where \( \lambda = \lambda(f) \) and \( \mu = \mu(f) \) are obtained as the roots of the following equations

\[
\mu \sinh \alpha \mu - \frac{(2\pi f)^2}{g} h_i \cosh \alpha \mu = 0,
\]

\[
\lambda \sinh \alpha \lambda - \frac{(2\pi f)^2}{g} h_i \cosh \alpha \lambda = 0.
\]

Having calculated the reflection coefficient \( K_r(f) \), the corresponding transmission coefficient is obtained from energy conservation as follows

\[
K_t(f) = \left(1 - K_r^2(f)\right) \frac{C_g(f)}{\left(C_g(f)\right)^2},
\]

where \( C(f), C_g(f) \) are the phase and group velocities, respectively, of the specific frequency component \( f \), in the region of incidence \( D^{(1)} \) (depth \( h_i \)) and in the regions of transmission \( D^{(3)} \) (depth \( h_i \)). The calculated reflection and transmission coefficients for the Roseau profile \( h_i = 30\text{m}, h_0 = 4\text{m}, \alpha = 15\pi / 180 \), are also plotted in Fig.2, in the frequency band \( f < 0.63\text{Hz} \).

4. OFFSHORE & NEARSHORE WAVE DATA

The data used in the present work are long-term time series of significant wave height that come from two sources. First, a thirty five year long time series of hindcast data, and, second, a sixteen year long series of buoy measurements; see Table 1. Hindcast data have kindly been provided by Prof. Lopatoukhin, St. Petersburg State University, Russia. Buoy data are freely available in the Internet through the site of the National Data Buoy Center, National Oceanic and Atmospheric Administration (NDBC/NOAA). The former series is complete, while the latter is incomplete. The percentage of missing values is 20%. After examining carefully the distribution of missing values per year (see Fig. 3), we finally decided to exclude three years (1995, 1997, 2000), because more than the half of the values that should be contained in these years are missing; see Fig. 3. Then, the total amount of missing values has been reduced to 12.4%, which significantly improved the extreme-value predictions.

<table>
<thead>
<tr>
<th>Basin</th>
<th>Lat</th>
<th>Long</th>
<th>Depth</th>
<th>Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baltic Sea</td>
<td>55.33N</td>
<td>20.55E</td>
<td>30</td>
<td>1954-1988</td>
</tr>
<tr>
<td>Gulf of Mexico</td>
<td>27.91N</td>
<td>95.36W</td>
<td>82.3</td>
<td>1990-2005</td>
</tr>
</tbody>
</table>
Using the above data as input to our methodology, described in the previous sections, long-term time series of nearshore wave data have been derived at a depth $h = 4m$. As an example, in Fig. 4 the bivariate $(H_s - T)$ plot of offshore wave data in the Baltic Sea (point: 55.33N, 20.55E, depth $h = 30m$) is shown, in comparison to the same plot concerning the transformed nearshore wave data at a depth $h = 4m$, using the Roseau shoaling profile shown in Fig. 2. The corresponding scatter diagrams of offshore vs nearshore values of the significant wave height and mean wave period are shown in Figs. 5 and 6, respectively.

5. EXTREME-VALUE ANALYSIS

In the present section, various methods for extreme-value analysis are applied to both offshore (input) and nearshore (derived) time series of significant wave height. The results are presented in the form of the well-known return period plots.

Both traditional and enhanced methods are implemented. More specifically, we have applied four variants of the well-known Gumbel’s approach, the POT method for various threshold values $u$, and a new method for calculating return periods from nonstationary time series (MENU method). The parameters of the extreme-value distribution in the Gumbel approach have been estimated using four different approaches: (i) the probability paper method, (ii) the least-squares return period relative error method, (iii) the probability paper method with Gringorten plotting positions, and (iv) the $L$-moments method. For the POT method, the following threshold values have been considered: $X_u = 3, 4, 5m$ (Baltic Sea) $X_u = 2, 3, 4m$ (Gulf of Mexico).

The MENU method, which is based on the MEan Number of Upcrossings of the level $x^*$, has recently been developed by Stefanakos and Athanassoulis (2006); see also Stefanakos and Monbet (2006). The MEan Number of Upcrossings of the level $x^*$ is given by
\[ M(x^*; t_1, t_2) = \frac{1}{2} \int_{-\infty}^{t_2} \int_{-\infty}^{t_1} |f_X(x^*, x)| \, dx \, dt. \] 

(5.1)

The return period \( T_R(x^*, t) \) associated with the level \( x^* \) and the starting time \( t \) is calculated as the unique value \( T \) for which

\[ M(x^*; t_1, t + T_R) = 1. \] 

(5.2)

Note also that, from relation (5.1) the form of the quantity \( M(x^*; t_1, t + T_R) \) is totally independent from the specific type of the underlying bivariate distribution considered. For the numerical implementation of the MENU method, the joint probability density function \( f_{XX}(x_1, x_2) \) is evaluated by introducing an appropriate nonstationary stochastic modelling of the time series \( X(\tau) \).

**Nonstationary stochastic modelling of time series**

A many-year long time series of, e.g., wind speed or significant wave height, can be treated as a nonstationary stochastic process, admitting of the following decomposition

\[ X(\tau) = G(\tau) + \sigma(\tau) W(\tau), \] 

(5.3)

where \( G(\tau) \) and \( \sigma(\tau) \) are deterministic periodic functions with period of one year, and \( W(\tau) \) is a zero-mean, stationary, stochastic process. The functions \( G(\tau) \) and \( \sigma(\tau) \) are called seasonal mean value and seasonal standard deviation, respectively, and are used to describe the exhibited seasonal patterns. Let it also be noted that, any possible long-term (climatic) trend, can be incorporated in the methodology, if the data permits us to identify such a trend.

This modelling has been first introduced in Athanassoulis & Stefanakos (1995); see also Stefanakos (1999), Stefanakos et al (2003), Stefanakos & Belibassakis (2005), Stefanakos et al (2006).

By differentiating Eq. (5.3), we can obtain the decomposition of \( X(\tau) \)

\[ \dot{X}(\tau) = \dot{G}(\tau) + \dot{\sigma}(\tau) W(\tau) + \sigma(\tau) \dot{W}(\tau). \] 

(5.4)

The functions \( \dot{G}(\tau) \) and \( \dot{\sigma}(\tau) \) are the derivatives of \( G(\tau) \) and \( \sigma(\tau) \) with respect to \( \tau \), and, by definition, are periodic.

Eqs (5.3) and (5.4) can be considered as a time-dependent transformation defining \( X(\tau) \) and \( \dot{X}(\tau) \) by means of \( W(\tau) \) and \( \dot{W}(\tau) \) (Stefanakos 1999). Thus, the problem is reduced to the evaluation of the time-invariant bivariate pdf \( f_{WW}(v_1, v_2) \). In the sequel, by considering that the process \( \dot{W}(\tau) \) is approximated using finite differences, the density \( f_{WW}(v_1, v_2) \) is calculated from the density \( f_{WWW}(u_1, u_2) \) (second-order density of \( W(\tau) \) at the time instances \( \tau \) and \( \tau + \Delta \tau \)), as follows:

\[ f_{WW}(v_1, v_2) = \Delta \tau f_{WWW}(v_1, v_1 + \Delta \tau, v_2), \] 

(5.5)

where \( \Delta \tau \) is the sampling interval. Finally, the pdf \( f^{ext}_{ss}(s_1, s_2) \) is calculated as follows

\[ f^{ext}_{ss}(s_1, s_2) = \frac{1}{\sigma^2} f^{WW}_{ss}\left(\frac{s_1 - g_{s1}}{\sigma_s}, \frac{s_2 - g_{s2}}{\sigma_s}, \frac{\sigma_s - (s_1 - g_{s1}) \sigma_r}{\sigma_r} \right). \] 

(5.6)

Thus, for the application of MENU method, two components are required:

- estimations of the seasonal mean value and seasonal standard deviation and their derivatives, and
- estimations of the parameters of the joint probability density function of \( W(\tau) \) and \( \dot{W}(\tau) \).

In Figs. 7 and 8, respectively, the estimates of the seasonal mean value (mMMV) and seasonal standard deviation (mMSD) for both datasets are shown.

Subsequently, using the Plackett model with univariate marginals Weibull for the density \( f^{WW}_{ss}(v_1, v_2) \), we can obtain the density \( f^{ext}_{ss}(s_1, s_2) \). Then, the return value estimates of both offshore and nearshore data are calculated and presented in Figs. 9 (Baltic Sea) and 10 (Gulf of Mexico). In these figures,
two distinct batches of curves are depicted: the higher represents the estimates based on the offshore data, while the lower corresponds to the nearshore estimates. The latter is narrower than the former, giving a greater stability to the derived estimates.

Inspecting all methods, we can conclude that there is an almost constant distance of approximately one meter between the offshore and nearshore predictions. Moreover, MENU gives lower estimates in both offshore and nearshore analysis than the traditional approaches. There is only one exception (nearshore, Gulf of Mexico) where MENU gives slightly higher results than the other methods, a fact that has to be examined in more detail.

**CONCLUSIONS**

In the present work a weakly nonlinear wave model, developed for the transformation of offshore wave systems to nearshore ones, is systematically applied to the derivation of long-term series of spectral wave parameters on decreasing depth from corresponding offshore wave data. The derived long-term series of nearshore wave parameters have been used as input to a new method, recently developed by Stefanakos & Athanassoulis (2006), for calculating return periods of various level values from nonstationary time series data.

The latter method is based on a new definition of the return period, that uses the MEan Number of Upcrossings of the level $x^*$ (MENU method), and it has been shown to lead to predictions that are more realistic than the traditional methods. To examine the effects of steep bottom topography on the nearshore extreme value predictions, Roseau (1976) bottom profiles have been used for which analytical expressions are available concerning the reflection and transmission coefficients. A parametric (JONSWAP) model is also used to synthesize offshore spectra from integrated parameters, which are then transformed on decreasing depth. Despite the fact that in the present work results are given only for 2D shoaling bottom topographies (parallel bottom contours), our model permits its extension to treat more realistic 3D bottom topographies.

The results of the extreme value analysis by the present method have been compared with corresponding ones obtained by traditional methods (Gumbel’s approach and POT method) and they are found to be in good agreement. In the examined cases, considering anyone of the above methods, we conclude that there is an almost constant distance of approximately one meter between the offshore and nearshore extreme value predictions. The latter could be important in various applications, such as the design of nearshore structures, and the calculation of long-term wave climatic impact on the coastal environment.

Figure 10. Return periods from (a) offshore and (b) nearshore time series using Gumbel, POT and MENU methods (Source: Buoy measurements, 27.91°N, 95.36°W, Gulf of Mexico).
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