A COUPLED-MODE MODEL FOR THE TRANSFORMATION OF WAVE SYSTEMS OVER INHOMOGENEOUS SEA/COASTAL ENVIRONMENT

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ABSTRACT

The transformation of the directional wave spectrum over an inhomogeneous sea/coastal environment is considered. Inhomogeneities include intermediate-water depth, strongly varying 3D bottom topography and ambient currents. The consistent coupled-mode model, developed by Athanassoulis and Belibassakis (1999), extended to three dimensions by Belibassakis et al. (2001) and applied to the transformation of wave systems over 3D bottom topography (Gerostathis et al 2008) is exploited for the calculation of the transfer function, connecting the incident wave with the wave conditions at each point in the field. This model is fully dispersive and takes into account reflection, refraction, and diffraction phenomena. In the present work, the coupled mode system is enhanced to account also for the effects of steady currents (Belibassakis et al, 2008), as well as, the effect of wave energy dissipation due to bottom friction and wave breaking. Numerical results obtained by the present model are compared with other models (as, e.g., Li et al 1993, Yoon et al 2004) and experimental measurements (Vincent and Briggs 1989), demonstrating the usefulness and practical applicability of the present method.

1. INTRODUCTION

In this work, the problem of transformation of the directional spectrum of an incident wave system over a region of strongly varying three-dimensional bottom topography is studied. Except of variable bathymetry also additional scattering effects due to the presence of ambient currents are included in the formulation. Also, wave dissipation due to bed friction and wave breaking is taken into account. The present approach is based on the utilization of the Coupled-Mode System (CMS), initially developed by Athanassoulis and Belibassakis (1999) in the case of waves propagating over variable bathymetry regions without the presence of currents, generalised to 3D by Belibassakis et al (2001), and applied to the transformation of wave systems over general bottom topography by Gerostathis et al (2008). The main feature of this model is the consistent calculation of the corresponding deterministic wave field, for each frequency, through which the transfer functions between two (possibly different) physical quantities of interest, at any two points in the variable bathymetry domain are obtained. The treatment of the monochromatic wave problems is obtained by formulating a diffraction problem on the basis of the decomposition of the depth into a parallel-contour surface and a perturbation bottom topography. Then, the 3D harmonic wave solution, for each frequency in the spectrum, is obtained as the superposition of the wave field excited by the obliquely incident harmonic waves over the background 2D topography and the 3D scattering field induced by the perturbation bathymetry, acting as localised scatterers. The main features of CMS are the exact calculation of the wave field (velocity and pressure field up to and including the bottom boundary), without any assumption concerning bottom slope and curvature, and its ability to reduce to simplified models, such as the modified mild-slope equation (Massel 1993, Chamberlain and Porter, 1995), at subregions where such a simplification is permitted.

In a recent work, Belibassakis et al (2008), the above CMS has been reformulated with respect to the total wave potential enabling us to include, except of 3D depth variations, also scattering effects by ambient current of general horizontal structure, with application to monochromatic incident waves. In the present work we extend the above analysis to wave incident systems characterised by a directional spectrum. In order to test our method, comparisons are presented concerning spectrum transformation with experiments by Vincent and Briggs (1989) concerning refraction/diffraction of irregular waves over an elliptic shoal. Although the present approach is based on the framework of linear theory, it still brings into light significant features as concerns the effects of seabed steepness and current scattering effects, permitting the transformation of directional wave spectra over a variable bathymetry region and the calculation of the spatial evolution of various quantities (free surface elevation, velocity, pressure etc) at every point in the domain. Also, it can be extended to treat weakly non-linear wave systems propagating in inhomogeneous environments.
2. DESCRIPTION OF THE ENVIRONMENT

We consider irregular wave propagation in a variable bathymetry region, characterized by a depth function $h(x_1, x_2)$ and in the presence of a steady (ambient) current $U(x)$; see Fig. 1. The current is assumed to be horizontal and slowly varying. Moreover, we assume that in the offshore region the current diminishes. The forcing comes from an incident wave system generated in the far up-wave region ($x_1 \to -\infty$), which is completely defined by the corresponding incident directional wave spectrum $S_{\text{inc}}(\omega, \theta)$. For computational purposes, in the present work, the directional wave spectrum is modeled as follows:

$$S_{\text{inc}}(\omega, \theta) = S_j(\omega; H, \theta) G(\omega, h_1) D(\theta, \theta_{\alpha}),$$  \hspace{1cm} (1)

where $S_j$ is the JONSWAP frequency spectrum.
\[ S_f(\omega; H, T_p) = \frac{ag^2}{\omega^2} \exp\left[-\frac{5}{4} \left(\frac{\omega}{\omega_p}\right)^{-2}\right] \gamma^\delta. \]  
\(2a)\)

In the above equation, \(\alpha\) is the Phillips constant, \(\omega_p = 2\pi / T_p\) is the peak angular frequency, \(\gamma^\delta\) is the peak enhancement factor with \(\delta = \exp\left[-\left(\omega - \omega_\alpha\right)^2 / \left(2\omega_\alpha^2\omega_p^2\right)\right]\), and \(G(\omega, h)\) is a filter function, defined by
\[ G(\omega, h_1) = 1 + \left[\frac{f^2(\omega)}{f^2(\omega_\alpha)}(1 + 2\alpha^2 f(\omega) / \sinh(2\alpha^2 f(\omega)))\right], \quad (2b) \]
where \(\omega_\alpha = \omega_\alpha h_1 / g\) and \(f(\omega) = \tanh^{-1}(k(\omega)h_1)\), modeling spectral saturation effects due to finite water depth (see, e.g., Massel 1989, Sec.7.3, and Massel 1996). Also, in Eq. (1), \(D(\theta; \theta_n)\) denotes the directional spreading function defined as follows:
\[ D(\theta; \theta_n) = \frac{1}{2\pi} + \frac{1}{\pi} \sum_{n=1}^{\infty} \exp\left[-i\frac{\sigma_n^2}{2}\cos\left[\frac{\theta - \theta_n}{\pi}\right]\right], \quad (3) \]
where \(\sigma_n\) is a parameter controlling the effective directional spreading (see, e.g., Li et al. 1993).

We consider a fixed (deterministic) bottom topography and no other free-surface forcing attributes, as e.g., wind-input, and thus, the random character of the wave system is induced only by the incident wave system. Under the additional assumption of waves of small steepness, small-amplitude (linear) wave theory can be used. We introduce however, no assumption as regards the mildness of the bottom surface. In this case, the essential feature of the random wave field is the strong spatial inhomogeneity. Moreover, we assume pointwise stationarity at any given point in the wave field, permitting us to employ transformation techniques for calculating the wave conditions in the domain.

A Cartesian coordinate system is used, having its origin at some point on the unperturbed free-surface \((z = 0)\). The \(z\)-axis is pointing upwards and the \(x\)-axis is pointing along the shoreward direction; see Fig. 1. The incident wave conditions are represented by the directional spectrum \(S_{\text{inc}}(\omega, \theta)\). Given the incident wave spectrum, the (free-surface elevation) spectra \(S(\omega, x)\), at each point \(x = (x_1, x_2)\) of the horizontal plane, can be obtained by spectral synthesis
\[ S(\omega, x) = \int_{-\infty}^{\infty} S_{\text{inc}}(\omega, \theta) \left| K(\omega, \theta; x) \right|^2 \, d\theta, \quad (4) \]
where \(K(\omega, \theta; x)\) denotes the appropriate transfer function defined by the harmonic wave solution in the area, corresponding to a given incident wave frequency \(\omega\) and direction \(\theta\); see, e.g., Goda (2000). The spectra associated with other physical quantities can be similarly obtained through the appropriate transfer functions.

In order to obtain the complete harmonic wave solutions and the transfer function, in previous work (Gerostathis et al. 2008) the coupled-mode system (CMS) developed by Belibassakis et al. (2001) has been used. In the latter approach, the depth \(h(x_1, x_2)\) is decomposed to \(h = h_1(x_1) + h_2(x_1, x_2)\), i.e. a parallel-contour surface \(h_1(x_1)\) and a perturbation bottom topography \(h_2(x_1, x_2)\). Then, the 3D harmonic wave solution, for each pair of frequency and direction in the spectrum, is obtained as the superposition of the wave field excited by the obliquely incident harmonic waves over the background topography \(h_1(x)\) and the 3D scattering potential induced by the perturbation bathymetry \(h_2(x, y)\). In a recent work Belibassakis et al. (2008) the above CMS has been reformulated with respect to the total wave potential, enabling us to include, except of the variable depth effects, also the scattering effects by ambient currents \(U(x)\) of general horizontal structure, characterized by relatively small variation lengths in comparison to the characteristic wavelength.

3. THE DETERMINISTIC CMS

By considering the wave velocity field, corresponding to each frequency and direction, to be time harmonic with absolute angular frequency \(\omega\), the potential is described by
\[ \Phi(x, z; t) = \text{Re} \left\{ \varphi(x, z) \exp\left(-i\omega t\right) \right\}, \quad (5) \]
where \(\varphi(x, z) = \varphi(x_1, x_2, z)\) is the complex wave potential and \(i = \sqrt{-1}\). In this case, the variational principle derived by Luke (1967), describing the kinematics and dynamics of the problem (see, e.g., Massel, 1989), after linearization, is written as follows:
\[ \int \int d\mathbf{x}\, d\mathbf{z} \left\{ \frac{\partial^2 \varphi}{\partial z^2} + \nabla \varphi \nabla h \right\} \left[ A + \frac{\partial^2 \varphi}{\partial z^2} \right]_{z=0} = 0, \quad (6) \]
where the term \(A\) on the mean free-surface \((z=0)\) is defined as follows (Belibassakis et al 2008):
\[ A = -i\omega^2 - 2i\omega(\mathbf{U} \cdot \nabla) - i\omega(\mathbf{U} \cdot \nabla) \mathbf{V} + \nabla \left( \mathbf{U} \left[ (\mathbf{U} \cdot \nabla) \varphi \right] \right), \quad (7) \]
and \(\nabla = (\partial / \partial x_1, \partial / \partial x_2)\) denotes the horizontal gradient.

Following previous works by the authors (see, e.g., Athanassoulis & Belibassakis 1999, Belibassakis et al 2001), the solution concerning the wave potential for each frequency in the incident spectrum is represented by a local-mode series
\[ \varphi(x, z) = \sum_{n=1}^{\infty} \varphi_n(x) \cdot Z_n(z; x), \quad (8) \]
where the functions \(Z_n(z; x), n = 0, 1, 2, \ldots\), are obtained as the eigenfunctions of local vertical Sturm-Liouville problems, formulated with respect to the local depth and the local intrinsic frequency
\[ \sigma = \omega - \mathbf{U} \cdot \mathbf{k}, \quad (9) \]
in the vertical interval \(-h(x) < z < 0\), and are given by
\[ Z_n = \frac{\cosh[k_n(z + h)]}{\cosh(k_n h)}, \quad Z_n = \frac{\cos[k_n(z + h)]}{\cos(k_n h)}, \quad n = 1, 2, \ldots \quad (10) \]
The wavenumbers \( k_n \) in Eq. (10) are obtained as a solution to the local dispersion relation associated with the intrinsic frequency:

\[
\sigma^2 = k_n g \tanh (kh) = -k_n g \tan (kh). \tag{11}
\]

Let it be noted here that, in the case of current, the direction of the wavevector \( \mathbf{K} \) is not known in advance and is determined through iterations. As concerns the additional mode \( \varphi_n Z_n \), appearing in the local-mode series, this is an extra term enabling the consistent satisfaction of the bottom boundary condition on a sloping bottom (Athanassoulis & Belibassakis 1999). A specific form of the sloping-bottom mode \( Z_n(x) \) is given by

\[
Z_n = h \left[ \frac{(z/h)^2}{(1+(z/h)^2)} \right],
\]

having the following properties: \( dZ_n(z=-h)/dz=1 \), \( Z_n(z=-h) = 0 \), and \( Z_n(z=0) = Z_n(z=0) = 0 \) (however other forms are also possible). Using the expansion (8), in conjunction with the properties of vertical eigenfunctions \( Z_n \), we obtain

\[
\frac{\partial \varphi_n}{\partial z} = \frac{\sigma^2}{g} \varphi_n, \quad \text{on} \quad z = 0. \tag{13}
\]

Introducing the above result, Eq. (13), in the last form of the variational principle, Eq. (6), we obtain

\[
\int \int dt \ dx \ dz \left[ \sum_{m,n} \left( a_{mn} \nabla^2 \varphi_m + \left( b_{mn} + \frac{2i\omega}{g} U \right) \cdot \nabla \varphi_m + \left( c_{mn} + \frac{\sigma^2 - \sigma_n^2}{g} \right) \varphi_m \right) + \frac{-1}{g} \nabla \cdot \left( U \left( \nabla \cdot \varphi_n \right) \right) \right] dz = 0, \quad m = -1,0,1,\ldots, \tag{15}
\]

The coefficients \( a_{mn}, b_{mn}, c_{mn} \) of the above CMS are all functions of the horizontal position \( x = (x_1, x_2) \) and given by

\[
a_{mn} = \langle Z_m, Z_n \rangle, \tag{16}
\]

\[
b_{mn} = 2 \langle \nabla Z_m, Z_n \rangle + Z_m(-h) Z_n(-h) \nabla h, \tag{17}
\]

\[
c_{mn} = \langle \nabla^2 Z_m, Z_n \rangle + \left( \frac{\partial^2 Z_m}{\partial z^2} \right)_{z=-h} + \langle \nabla Z_m, \nabla h \rangle Z_n(-h), \tag{18}
\]

where the bracket denotes inner product in the vertical interval

\[
\langle f, g \rangle = \int_{z=-h}^{z=0} f(z) g(z) \, dz.
\]

An important feature of the calculation of the wave fields by means of the enhanced local-mode representation (8), is that it exhibits an improved rate of decay of the modal amplitudes \( \varphi_n \) of the order \( O(n^{-4}) \). Thus, only a small number of modes suffice to obtain a convergent solution, even for bottom slopes above 100%. In addition, a significant simplification of the above CMS (15) is obtained by keeping only the propagating mode \( \varphi_0(x) \) in the local-mode series expansion of the wave potential, which is the term that essentially describes the propagation features, leading to a new one-equation model. Using the expressions of the coefficients (16,17,18) for \( m=n=0 \) in the above one-equation model (Eq. 15 for \( m=0 \), using \( \varphi_0 = 0, n = -1,1,2,\ldots \)) and multiplying the result by \( g \), we derive the equivalent form:

\[
\nabla \left( C \nabla \varphi_0 \right) - \nabla \cdot \left( U \left( \nabla \cdot \varphi_0 \right) \right) + 2i\omega U \cdot \nabla \varphi_0 + \left[ k_0^2 C + g c_{00}^{(2)} + \omega^2 - \sigma^2 + i\omega (\nabla \cdot \mathbf{U}) \right] \varphi_0 = 0, \tag{19}
\]

where the coefficient \( c_{00}^{(2)} = \langle \nabla^2 Z_0, Z_0 \rangle + \nabla h Z_0(-h) \) contains terms proportional to first and second horizontal derivatives of the depth function (proportional to bottom slope and curvature), as well as first and second horizontal derivatives of the horizontal current velocity components \( U_1 \) and \( U_2 \). It can be easily seen that, in the case of no current (\( U = 0 \)), the above equation (19) exactly reduces to the simplified Modified Mild-Slope model (MMS) developed by Massel (1993) and Chamberlain & Porter (1995). Furthermore, if the term \( c_{00}^{(2)} \) (associated with second-order effects of bottom slope and curvature, as well as current variations) is approximately omitted, then Eq. (19) reduces to Kirby’s (1984) mild slope equation that includes the effect of background currents.

4. WAVE ENERGY DISSIPATION

The coupled-mode system (15) deals with ideal (inviscid) flow. However, the dissipation of wave energy due to bottom friction and wave breaking can be indirectly taken into account, by including an appropriate imaginary part in the wavenumber, similarly as in the case of the mild-slope and the extended mild-slope models; see, e.g., Dingemans (1997), Massel (1992). Since the propagating mode \( \varphi_0(x) \) is the term that essentially carries out the wave propagation features, in order to model energy dissipation, the coupled-mode system is modified as follows

\[
\sum_{m,n} \left( a_{mn} \nabla^2 \varphi_m + \left( b_{mn} + \frac{2i\omega}{g} U \right) \cdot \nabla \varphi_m + \left( c_{mn} + \frac{\sigma^2 - \sigma_n^2}{g} \right) \varphi_m \right) + \frac{-1}{g} \nabla \cdot \left( U \left( \nabla \cdot \varphi_n \right) \right) \right] dz = 0, \quad m = -1,0,1,\ldots, \tag{20}
\]

where \( \delta_{mn} \) is Kronecker’s delta and \( \gamma \) denotes the dissipation coefficient

\[
\gamma(x) = \gamma_1(x) + \gamma_2(x),
\]

combining the effects of seabed friction and wave breaking on wave propagation; see, e.g., Massel (1992). The bottom friction dissipation coefficient \( \gamma_2(x) \) is obtained in terms of the local

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wave-velocity at the bottom, as follows (Massel & Gourlay, 2000),

$$\gamma_f(x) = \frac{16f_o}{3\pi g C_s(x)} \left| \frac{u_b(x)}{H^2(x)} \right|,$$  \hspace{1cm} (22)

where $u_b$ denotes the bottom orbital velocity, and $H(x)$ is the local wave height. In Eq. (22), $C_s$ is the local wave group velocity, and for turbulent flow, the bottom friction coefficient $f_o$ is dependent on the bottom surface roughness, taking values in the interval $0.1 < f_o < 0.2$ in natural conditions. The latter is of order of 0.01 for sandy bottoms; see e.g., Dingemans (1997). Following Massel & Gourlay (2000), the wave breaking dissipation coefficient $\gamma_b$ is modeled as follows:

$$\gamma_b(x) = \frac{\alpha \sigma}{\pi} \sqrt{gh(x)} H(x),$$ \hspace{1cm} (23)

and zero otherwise. In the above equation $C = \sigma \kappa$ is the local phase velocity, $H_{max}$ is the local maximum allowable wave height, and $\alpha = O(1)$ is an empirical coefficient. In the present work, the expressions suggested by Massel & Gourlay (2000) have been used to determine the subregions where breaking occurs,

$$\frac{H_{max}}{h} = \begin{cases} \frac{0.937}{{\sqrt h}^{0.155}} \left( \frac{k_bH}{2\pi} \right)^{-0.130}, & \text{when } |\nabla h| > 0.025 \\ 1 + \frac{1}{0.01504h^{1.5}}, & \text{when } |\nabla h| \leq 0.025 \end{cases},$$  \hspace{1cm} (24)

where $h = h/gT^2$ and $T$ and $H$ denote the incident wave-period and waveheight, respectively.

As concerns the empirical coefficient $\alpha$, it is taken to be given in terms of the bottom slope and the non-linearity parameter $F_{ro} = (H/h_{max})^{1/2} (g/2h_{max})^{1/2}$, based on the incident wave height $H$, by the following formula,

$$\alpha = (0.45 + 0.1|\nabla h|) \sqrt{(F_{ro} - 100)}/380, \text{ when } F_{ro} > 100,$$  \hspace{1cm} (25)

and zero otherwise. The latter has been found to model various data in cases with moderate and large bottom slopes examined by Massel & Gourlay (2000). We note here that, since the actual waveheight in the domain is not known in advance, both the dissipation factor and the final result concerning the wave potential and the rest quantities of interest are obtained by iterations.

For the numerical solution the system of equations (20) is supplemented by appropriate boundary conditions expressing the incoming wave, at the specific direction for each frequency, along the offshore boundary $x = a_1$. Furthermore, all along the lateral boundaries $(x = x_1)$ and the shoreward boundary $(x = a_2)$ absorbing boundary conditions are imposed, using the Perfectly Matched Layer (PML). More details concerning the implementation of the PML are given in Belibassakis et al (2001).

5. CALCULATION OF THE TRANSFER FUNCTIONS

Having calculated the complex wave potential $\varphi(x, z)$ for each single frequency $\omega$ and direction $\theta$ in the incident spectrum, as provided by the solution of CMS, the complex amplitude of other physical quantities of interest, in the domain, can be calculated. For example, the free-surface elevation is obtained as follows

$$\eta(x, \omega, \theta) = \frac{1}{g} \left( i\omega \frac{\text{UV} \varphi(x, z = 0)}{\varphi(x, z = 0)} \right) \varphi(x, z = 0),$$  \hspace{1cm} (26)

the horizontal and vertical wave velocity components by

$$\mathbf{u}(x, z; \omega, \theta) = \nabla_{3D} \varphi(x, z),$$  \hspace{1cm} (27)

where $\nabla_{3D} = \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right)$ is the 3D gradient operator, and the (dynamic) pressure by

$$p(x, z; \omega, \theta) = \left( i\omega - \frac{\varphi(x, z)}{\varphi} \right) \varphi = i\sigma \varphi,$$  \hspace{1cm} (28)

where $\rho$ denotes the fluid density. Then, on the basis of linear system theory applied to our distributed system, the transfer function between any two quantities at any two points is obtained as the ratio of the corresponding complex amplitudes. For example, the transfer function between anyone of the above quantities (compactly denoted as $M$) and the free-surface elevation associated with the incident wave system is

$$K(\omega, \theta; x, z) = \frac{M(x, z; \omega, \theta)}{H/2},$$  \hspace{1cm} (29)

where $H$ is the waveheight of each spectral component in the incident spectrum. In this way, the spatial distribution of the spectrum associated with each quantity $M$, is expressed as

$$S_h(\omega, x_1, x_2, z) = \int_{-\pi/2}^{\pi/2} S_{2D}(\omega, \theta) |K(\omega, \theta; x, z)|^2 d\theta,$$  \hspace{1cm} (30)

and the corresponding moments are obtained by frequency integration,

$$m_v(x, z) = \int_{-\pi/2}^{\pi/2} \omega^n S_h(\omega, x, z) d\omega, \text{ v=0,1,2,...}.$$  \hspace{1cm} (31)

Detailed results concerning the application of the above method in the case normally incident wave systems propagating over 1D bottom profiles, without effects of currents, have been presented in Athanassoulis et al (2003), and in the case of 3D bottom topographies in Gerostathis et al (2008), where also comparisons are presented vs. measured data and the predictions by phase-averaged models like SWAN (see Ris et al 1999).

6. NUMERICAL RESULTS AND DISCUSSION

In order to evaluate the results obtained by the present method, in this section comparisons are presented concerning transformation of wave fields and spectral parameters vs.
Fig. 2: Refraction pattern above the elliptic shoal without current for waves of period $T=1.3s$ at $\theta=0^\circ$ incidence.

Fig. 3: Spatial distribution of the mean flow due to breaking of waves inducing strong modification to the refraction pattern.

Fig. 4: Calculated wave field in the case N4 without current, for harmonic incident waves of period $T=1.3s$ at $\theta=0^\circ$ incidence.

Fig. 5: Calculated wave field in the case N4 without current, for harmonic waves of period $T=1.3s$ at $\theta=15^\circ$ incidence.

Fig. 6: Calculated wave field in the case N5 with the effects of current, for harmonic waves of period $T=1.3s$ at $\theta=0^\circ$ incidence.

Fig. 7: Calculated wave field in the case N5 with the effects of current for harmonic waves of period $T=1.3s$ at $\theta=15^\circ$ incidence.
laboratory data. In particular, we consider experimental data provided by Vincent and Briggs (1989) in the case of refraction/diffraction of irregular waves over an elliptic shoal, without any flow, and in the presence of current generated by the breaking of water waves passing over the elliptic shoal.

More specifically, the cases N4 and N5 of Vincent and Briggs (1989) have been simulated. In both cases the peak frequency of the incident wave spectrum is defined to be: $T_p = 1.3s$, the peak enhancement factor $γ = 20$ and the directional spreading parameter $σ_θ = 10^°$. Thus, the spectral wave components essentially lie in the directional interval $-30^° < θ < 30^°$.

In the first case (N4) examined, the significant waveheight of the incident wave spectrum is defined to be: $T_p = 1.3s$, the peak enhancement factor $γ = 20$ and the directional spreading parameter $σ_θ = 10^°$. Thus, the spectral wave components essentially lie in the directional interval $-30^° < θ < 30^°$.

In the second case (N5) the significant waveheight of the incident spectrum is much greater, of the order of $H=16cm$ in front of the scatterer (and $H=19cm$, $α=0.0262$ at the wavemaker). In this case, due to significant breaking over and behind the scatterer, a mean flow is generated, as presented and discussed also in Yoon et al (2004). In the present work this breaking-induced current has been approximated as illustrated in Fig.3, based on data from the latter reference, which were obtained by using the SHORECIRC model (van Dongeren et al 1994). As schematically presented in Fig.3, the above mean current defocuses the converging of wave rays at the rear part of the shoal, redirecting rays outwards from the center line, and producing significant modification on the refraction pattern and the spatial distribution of wave energy.

As an example, in Figs. 4 and 5 we present (using colorplots) the snapshots of the calculated wave field $φ(x,z=0)$ on the surface, normalized with respect to the amplitude of the incident wave, in the case of monochromatic waves of period $T=1.3s$ (equal to the peak period of the incident spectrum) and for $θ=0^°$ and $θ=15^°$ incidence angles, without the effects of current. The bathymetry corresponds to the elliptic shoal of Vincent and Briggs (1989) and the depth varies from $h=0.457m$ (the base of the scatterer) to $h=0.1522m$ (at the top of the elliptic shoal $x_r=12.5m$, $x_c=0$). Numerical results have been obtained by the present model using 5 totally modes ($m=1,0,1,2,3$) and discretizing the CMS using second-order finite differences based on 201X201 horizontal grid resolution, which was found enough for numerical convergence. We are able to observe in this figure the strong focusing of the wave energy, concentrated in an area about 1-1.5m downwave the elliptic shoal, in agreement with the predictions of ray theory (Fig.2).

In Figs. 6 and 7 we present corresponding colorplots of the calculated wavefield on the surface, in the case N5, with breaking-induced current flowing over the region of the elliptic shoal (as shown in Fig.3). In this case, the maximum current speed is of the order of 0.3m/s. Again, indicative results are presented for incident monochromatic waves of period $T=1.3s$, for $θ=0^°$ (Fig.6) and $θ=15^°$ (Fig.7) angles of incidence. We observe in this case that the current contributes to spreading the area of wave energy concentration several meters in the downwave direction from the scatterer. At the same time, the maximum wave amplitudes in the focal area are smaller than the ones in the same condition(s) without the current.

Finally, in Figs. 8 and 9 the distribution of significant wave height behind the elliptic scatterer is compared vs. experimental data, in the cases N4 and N5 of Vincent and Briggs (1989), respectively. Results are presented along section 4 (normal to the centerline downwave the elliptic shoal), indicated also in Figs. 2 and 3 by using dashed lines. Also, in these figures comparison with the theoretical results provided by Li et al (1993, only for N4 case) and Yoon et al (2005) are included, respectively. Present method numerical predictions are plotted using a thick solid line and have been obtained by spectral synthesis, through Eqs. (30) and (31) concerning the free-surface elevation, using a grid of 21 frequencies and 21 directions to discretize the incident wave spectrum (Eq. 1).
Comparing the results obtained in the cases N4 (Fig.8) and N5 (Fig.9), we conclude that the (breaking-induced) current significantly changes the spatial distribution of the energy pattern along Sec. 4, and this is also true for an extended area downwave the shoal. As concerns the computing time required by the present model for calculating the transfer function (corresponding to 441 combinations of incident monochromatic fields), this is of the order of 3h in the case of the one equation model Eq.(19), and 50h in the case of CMS Eq.(15), keeping 5 totally modes in the expansion of the wave potential (8), at a PC (Intel Dual Core @2.53GHz). However, we wish to stress here that the above calculation time can be significantly reduced by increasing the number of available processors (or nodes) and parallelizing the code, which is quite feasible since the calculation of the monochromatic fields (involved in the estimation of the transfer function) are independent.

The good agreement between our numerical predictions and experimental data, in the examined cases, is an indication that the present method is able to provide reasonable results concerning the transformation of irregular waves over inhomogeneous sea/coastal environment. Future work is planned towards the systematic evaluation of the present model in real cases concerning nearshore sites, possibly characterised by complex bathymetric terrains and other inhomogeneities, for which measured data are available.

CONCLUSIONS
The problem of transformation of the directional spectrum, associated with an incident wave system, over a region of strongly varying three-dimensional bottom topography and in the presence of ambient currents is studied using a novel coupled-mode system. In the examined case, environmental inhomogeneities are considered to be rapidly varying within distances comparable to the characteristic wave length, and thus, phase-averaged models are not expected to provide details of the strong scattering field due to depth and current variations. The wave analysis has been performed using the phase-resolving, Coupled Mode System presented by Athanassoulis and Belibassakis (1999), and extended to 3D by Belibassakis et al (2001), reformulated with respect to the total wave potential and incorporating the effects of scattering by steady currents. This model takes into account reflection, refraction and diffraction phenomena both due to bathymetric and current variations, and enables the calculation of the transfer function, connecting the incident wave with the wave conditions at each point in the field. The latter is then used to obtain the spatial evolution of point spectra over variable bathymetry regions, permitting the estimation of all interesting wave quantities (free surface elevation, velocity, pressure etc), at every point in the domain. Good agreement between calculations and experimental data has been demonstrated for specific cases considered, indicating the practical applicability of the present method to the transformation of irregular waves over inhomogeneous sea/coastal environment. Finally, the present model can be extended to treat weekly non-linear waves by calculating the corresponding quadratic transfer functions (first results in this direction have been presented in Belibassakis & Athanassoulis 2002), and it can be further coupled with solvers of the mean flow equations (as, e.g., presented in Belibassakis et al 2007) for modeling and studying wave propagation and wave-induced flow over general 3D bottom topography.

REFERENCES

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