HYDROELASTIC ANALYSIS OF VERY LARGE FLOATING BODIES OVER VARIABLE BATHYMETRY REGIONS

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Abstract. The coupled-mode model developed by Belibassakis & Athanassoulis (2005) is extended and applied to the hydroelastic analysis of three-dimensional large floating bodies of shallow draft or ice sheets of small thickness, lying over variable bathymetry regions. A general bathymetry is assumed, characterised by a continuous depth function, joining two regions of constant, but possibly different, depth. Following previous works for the propagation and diffraction of water waves over three-dimensional bathymetric terrains (Belibassakis et al 2001, Gerostathis et al 2008), we consider the scattering problem of harmonic incident surface waves, under the combined effects of variable bathymetry and a floating elastic plate of orthogonal planform shape. Under the assumption of small-amplitude waves and small plate deflections, the hydroelastic problem is formulated within the context of linearised water-wave and thin elastic-plate theory. In order to consistently treat the wave field beneath the elastic floating plate, down to the sloping bottom boundary, a complete, local, hydroelastic-mode series expansion of the wave field is used, enhanced by an appropriate sloping-bottom mode. The latter enables the consistent satisfaction of the Neumann bottom-boundary condition on a general topography. Numerical results concerning floating structures over flat and inhomogeneous seabeds are presented, and the effects of wave direction, bottom slope and bottom corrugations on the hydroelastic response are discussed.

1 INTRODUCTION
The interaction of free-surface gravity waves with floating deformable bodies, in water of intermediate depth and over a general bathymetry, is a mathematically interesting problem finding important applications. Very Large Floating Structures (VLFS, megafloats) and platforms of shallow draft are examples of structures for which hydroelastic effects are significant and should be properly taken into account. Such structures have been intensively studied, being under consideration for use as floating airports and mobile offshore bases. Extended surveys, including literature review, have been recently presented by Kashiwagi (2000) and Watanabe et al (2004). Also, the hydroelastic analysis of floating bodies is very relevant to problems concerning the interaction of water waves with ice sheets; see the extended reviews by Squire et al (1995) and Squire (2008).

The linearised problem associated with the hydroelastic responses of VLFS can be effectively treated in the frequency domain, and many methods have been developed for its solution. These include the B-spline Galerkin method by Kashiwagi (1998), Boundary Element Methods (Ertekin & Kim, 1999, Hermans, 2000, Hong et al, 2001), hydroelastic eigenfunction expansion techniques (Kim & Ertekin 1998, Takagi et al 2000, Hong et al 2003), integro-differential equations (Adrianov & Hermans, 2003), Wiener-Hopf techniques, (Tkacheva, 2001), Green-Naghdi models (Kim & Ertekin, 2002), and others. Another approach, originally developed by Eatock Taylor & Waite (1978) and Bishop et al (1986), and further extended by various authors, as e.g., Newman (1994), Wu et al (1995), is based on expressing the structure oscillations in a series expansion (using either dry elastic modes or another basis), identifying appropriate radiation problems and, finally, formulating and solving the coupled hydrodynamic equations. Moreover, Meylan (2001) derived a variational equation for the plate-water system by expressing the water motion as an operator equation. In addition to the above, high-frequency asymptotic methods have been developed to describe the deflection dynamics of VLFS, see, e.g., Ohkusu &
Numerical methods for predicting the hydroelastic responses of VLFS in variable bathymetry regions have been recently proposed, based on BEM in conjunction with fast multipole techniques (Utsunomiya et al., 2001), and on eigenfunction expansions in conjunction with step-like bottom approximation (Murai et al., 2003). In addition, Porter & Porter (2004) have derived an approximate, vertically-integrated, two-equation model for the problem of water wave interaction with an ice sheet of variable thickness, lying over variable bathymetry, which is valid under mild-slope assumptions both with respect to the wetted surface of the ice sheet and the bottom boundary. In the case of hydroelastic behaviour of large floating deformable bodies in general bathymetry, a new coupled-mode system has been derived and examined by Belibassakis & Athanassoulis (2005) based on local vertical expansion of the wave potential in terms of hydroelastic eigenmodes, and extending previous similar approach for the propagation of water waves in variable bathymetry regions (Athanassoulis & Belibassakis 1999). Similar approaches with application to wave scattering by ice sheets of varying thickness have been presented by Bennets et al (2007) based on multi-mode expansion.

In the present work, the coupled-mode model developed by Belibassakis & Athanassoulis (2005) is extended and applied to the hydroelastic analysis of 3D large floating bodies of finite extent and shallow draft or ice sheets of small thickness, lying over variable bathymetry regions. A general bathymetry is assumed, characterised by a continuous depth function, joining two regions of constant, but possibly different, depth. Following previous works for the propagation and diffraction of water waves over 3D bathymetric terrains (Belibassakis et al. 2001, Gerostathi et al. 2008), we consider the scattering problem of harmonic, obliquely-incident surface waves, under the combined effects of variable bathymetry and a floating elastic plate of orthogonal planform shape. Under the assumption of small-amplitude waves and small plate deflections, the hydroelastic problem is formulated within the context of linearised water-wave and thin elastic-plate theory. In order to consistently treat the wave field beneath the elastic floating plate, down to the sloping bottom boundary, a complete, local, hydroelastic-mode series expansion of the wave field is used, enhanced by an appropriate sloping-bottom mode. The latter enables the consistent satisfaction of the Neumann bottom-boundary condition on a general topography. First numerical results concerning floating structures lying over flat and inhomogeneous seabeds are comparatively presented, and the effects of wave direction, bottom slope and bottom corrugations on the hydroelastic responses are discussed.

2 FORMULATION

The studied environment consists of a water layer bounded above partly by the free surface and partly by a large floating elastic body (large shallow-draft platform or ice sheet of uniform and small thickness), and below by a rigid bottom; see Fig.1. It is assumed that the bottom surface exhibits an arbitrary variation in a bounded subdomain, which includes the support of the bottom inhomogeneity and the floating plate acting as localized scatterer(s). Outside this area, the bathymetry is characterised by parallel, straight bottom contours lying between

![Figure 1](image-url). Floating elastic body of length $L$ and breadth $B$ in variable bathymetry region, modelled as thin plate. The horizontal plane is decomposed into the plate region (E) and the water region (W) outside the rectangular shaped floating structure.
two regions of constant but different depth, \( h = h_1 \) (region of incidence) and \( h = h_2 \) (region of transmission). The wave field is excited by a monochromatic plane wave of angular frequency \( \omega \), propagating with an oblique direction \( \theta_0 \) with respect to the bottom contours. A Cartesian coordinate system is introduced, with its origin at some point on the mean elastic-plate surface (in the variable bathymetry region), the \( z \)-axis pointing upwards and the \( y \)-axis being parallel to the bottom contours. The function \( h = h(x, y) \), appearing in the above definitions, represents the local depth, measured from the mean water level. We consider the scattering problem of harmonic, obliquely-incident, surface (gravity) plane waves of angular frequency \( \omega \), under the combined effects of variable bathymetry and the finite thin, floating elastic plate, modelling a structure of orthogonal shape with length \( L \) and breadth \( B \), floating in the variable bathymetry region, as shown in Fig. 2. However, our analysis can be further extended to floating elastic structure of more general shape. Under the usual assumptions of linearised water-wave theory and thin elastic plate theory, the problem can be treated in the frequency domain. The wave potential is expressed in the following form

\[
\Phi(x, y, z; t) = \text{Re}\{\phi(x, y, z) \exp(-i\omega t)\},
\]

(1a)

where \( i = \sqrt{-1} \). The complex amplitude of the free-surface elevation \((\eta)\) is obtained in terms of the wave potential as follows

\[
\eta(x, y) = i \frac{\partial \phi(x, y, z = 0)}{\omega} \text{ } ,
\]

(2a)

where \( g \) is the acceleration due to gravity. In the area of the elastic-plate, the deflection \((w)\) is connected with the wave potential by a similar relation derived from the kinematical condition at the liquid-solid interface,

\[
w(x, y) = i \frac{\partial \phi(x, y, z = 0)}{\omega} \text{ } .
\]

(2b)

The differential formulation of the studied problem consists of the Laplace equation in the water layer

\[
\left(\nabla^2 + \frac{\partial^2}{\partial z^2}\right)\phi = 0, \quad \text{in} \quad -h(x, y) < z < 0, \quad (x, y) \in W \cup E ,
\]

(3a)

where \( \nabla = \left(\partial_x, \partial_y, \partial_z\right) \) denotes the horizontal gradient operator. On the part of the horizontal plane associated with the free surface the wave potential satisfies the linearized free-surface boundary condition

\[
\partial_z \phi - \mu \phi = 0 \text{ } , \quad \text{on} \quad z=0, \quad (x, y) \in W .
\]

(3b)

where \( \mu = \omega^2 / g \) the frequency parameter. Moreover, for points on the plate the wave potential satisfies the corresponding dynamical equation forced by the water pressure

\[
\nabla^2 \left(\frac{d}{\partial t^2} w\right) + (1-\varepsilon)w = i \frac{\mu}{\omega} \phi(x) \text{ } , \quad \text{on} \quad z = 0, \quad (x, y) \in E .
\]

(3c)

In addition, at the sea bottom the wave potential satisfies the no-entrance boundary condition

\[
\partial_z \phi - \eta h \nabla \phi = 0 \text{ } , \quad \text{on} \quad z = -h(x, y) .
\]

(3d)

Finally, at the plate edges the following two conditions apply

\[
\frac{\partial^2 w}{\partial n^2} + (2-\nu)\frac{\partial^3 w}{\partial n \partial s^2} = 0 \text{ } , \quad \text{at} \quad (x, y) \in \partial W \text{ } ;
\]

(3e)

\[
\frac{\partial^2 w}{\partial n^2} + \nu \frac{\partial^3 w}{\partial s^3} = 0 \text{ } , \quad \text{at} \quad (x, y) \in \partial W \text{ } ;
\]

(3f)

where \( n \) and \( s \) denote the normal and tangential derivatives, respectively. Boundary condition (3c) describes the coupled dynamics of the thin elastic plate (modelling the floating structure) and the underlying fluid flow; see, e.g., Meylan & Squire (1994) or Andrianov & Hermans (2003). It is obtained by combining the thin elastic plate equation with the linearised Bernoulli’s equation for the dynamic water pressure on the elastic plate surface, and involves the (constant) parameters \( d = D \rho g \) and \( \varepsilon = m \omega^2 \rho g \), where \( D = E t^3 / 12(1-\nu^2) \) denotes the flexural rigidity of the elastic plate (the equivalent flexural rigidity of the platform), where \( t \) denotes the thickness and \( \nu \) the Poisson’s ratio. Moreover, \( \rho \) denotes the fluid density and \( m \) is the mass per unit area of the plate. Finally, the edge conditions (3e,f) state that the ends of the plate are free of shear force and moment, respectively.

### 3 MODAL EXPANSION OF THE WAVE POTENTIAL

The studied problem, Eqs.(3), combines the character of water wave propagation and scattering in inhomogeneous bathymetric terrains, under the additional effects due to the presence of localized hydroelastic
scatterer (E). This type of problems have been extensively studied in a series of works by the authors, starting with the linearised water wave problem in general bathymetry (Athanassoulis & Belibassakis 1999, Belibassakis et al 2001), where the following local mode series expansion is used to consistently represent the wave field in the water region:

$$\varphi(x,z) = \varphi_n(x)Z_n(z;x) + \sum_{n=0}^{\infty} \varphi_n(x)Z_n(z;x), \quad -h(x) < z < 0, \quad x = (x, y).$$

(4)

The major part of the set of vertical modes $\{Z_n(z;x), n=0,1,2,\ldots\}$ is dependent on $x$ through $h(x)$ and is obtained as the solution of a vertical eigenvalue problem, formulated at each horizontal position. Moreover, $\varphi_n(x)Z_n(z;x)$ is an appropriate term, called the sloping-bottom mode, accounting for the satisfaction of the bottom boundary condition on the non-horizontal parts of the bottom. The idea of the sloping bottom mode, in conjunction with the above type of modal expansion, has been first introduced in the case of water waves propagating in variable bathymetry. Then, it has been used for many problems exhibiting similar features, such as nonlinear water waves (Athanassoulis & Belibassakis 2007, Belibassakis & Athanassoulis 2002, 2011), hydroacoustics (Athanassoulis et al 2008) and hydroelastic applications in variable bathymetry regions, formulated in the context of classical thin plate theory (Belibassakis & Athanassoulis 2005) and high-order shear deformable plate theory (Athanassoulis & Belibassakis 2009). In accordance with the latter works, the infinite set $\{Z_n(z;x), n=0,1,2,3,\ldots\}$ of functions describing the vertical structure of each mode, at each horizontal position $x$, are generated by

$$\begin{align*}
\partial_z^2 Z_n(z) - \kappa_n^2 Z_n(z) &= 0, \quad \text{in the vertical interval } -h(x) < z < 0, \quad (5a) \\
\partial_z Z_n(z=-h) &= 0, \quad \text{at the bottom } \quad z=-h(x), \quad (5b) \\
\alpha \partial_z Z_n(z=0) - \mu Z_n(z=0) &= 0, \quad \text{at the water-elastic body interface } \quad z=0, \quad (5c)
\end{align*}$$

where $\alpha(\kappa)$ is a function of $\kappa$ for hydroelastic waves and simplifies to $\alpha=1$ in the case of water waves. The solution the above local vertical eigenvalue problem is given by

$$Z_n(z) = \cosh^{-1}(\kappa_n h) \cosh[\kappa_n (z+h)], \quad n=0,1,2,3,\ldots, \quad (6)$$

where the eigenvalues $\{\kappa_n, n=0,1,2,\ldots\}$ are obtained as the roots of the (local at any horizontal position $x$) dispersion relation

$$\mu h = \alpha(\kappa) \kappa h \tanh(\kappa h), \quad \text{with } \alpha=1, \text{ for } x \in W, \text{ and and } \alpha(\kappa) = D \kappa^4 + 1 - \varepsilon, \text{ for } x \in E. \quad (7)$$

The distribution of the roots $(\kappa^W_n, \text{ for } x \in W$ and $\kappa^E_n, \text{ for } x \in E)$ of the above dispersion relation on the complex $\kappa$-plane, that are used in the expansion (4) are schematically plotted in Fig.2. For water waves the first root is real and is associated with the propagating mode and the rest are imaginary associated with the evanescent modes. For hydroelastic waves except of the previous categories there exist also roots on the complex plane associated with modes characterized by mixed propagating-evanescent character. Except of the propagating and evanescent modes the local mode expansion is augmented by the slopping bottom mode $\varphi_n(x)Z_n(z;x)$ permitting the consistent satisfaction of the Neumann boundary condition at the non-horizontal parts of the bottom surface and making the local mode series rapidly convergent. A specific convenient form of

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2.png}
\caption{Distribution of the roots of Eq. (15b) on the complex $\kappa$-plane.}
\end{figure}
the vertical structure $Z_{z}(z;h(x))$ of this mode is given by low degree polynomials, however, other forms are also possible. More details concerning the role and significance of this term can be found in Athanassoulis & Belibassakis (1999) and Belibassakis et al (2001).

4 COUPLED-MODE SYSTEM OF EQUATIONS

The coupled system of horizontal differential equations (CMS) is obtained by means of a variational principle developed by Belibassakis Athanassoulis (2005), using the representation of the local mode series expansion (4) of the wave potential in the water column below the free surface and below the elastic plate modelling the large elastic floating body. This permits us to reformulate the problem, Eqs.(3) with respect to the unknown modal amplitudes $\phi_n(x), n = -1, 0, 1, 2, \ldots$. for $x \in W \cup E$. The present CMS takes the following form

$$\sum_{n=-1}^{\infty} a_{mn}(x) \nabla^2 \phi_n(x) + b_{mn}(x) \nabla \phi_n + c_{mn}(x) \phi_n(x) = i \omega w(x) \cdot \chi(E), \quad m = -1, 0, 1, \ldots,$$

where $\chi(E)$ denotes the characteristic function of the plate subdomain $E$ (i.e., $\chi(E) = 1$, for $x \in E$, and 0 otherwise). The system (8) is supplemented by the following equation providing the coupling between the wave modes ($\phi_n$) and the elastic plate deflection ($w$):

$$\nabla^2 (d \nabla^2 w) + (1-\varepsilon) w = \frac{i \mu}{\omega} \sum_{n=1}^{\infty} \phi_n(x).$$

In the above equations, the horizontally-dependent coefficients $a_{mn}(x)$, $b_{mn}(x)$ and $c_{mn}(x)$ are given by the following expressions

$$a_{mn}(x) = \langle Z_n, Z_m \rangle,$$  \hspace{1cm} (10a)

$$b_{mn}(x) = 2\langle \nabla Z_n, Z_m \rangle + \nabla h \cdot Z_m(z=-h)Z_n(z=-h),$$  \hspace{1cm} (10b)

$$c_{mn}(x) = \langle \nabla^2 Z_n + \nabla Z_n \nabla Z_m, \nabla Z_n \rangle + \frac{\partial Z_n(z=-h)}{\partial z} + \nabla h \cdot \frac{\partial Z_n(z=-h)}{\partial \xi} Z_m(z=-h),$$  \hspace{1cm} (10c)

where $\langle f, g \rangle = \int_{z=-h(x)}^{z=0} f(z) g(z) dz$. After solving the CMS, Eqs. (8), (9), the wave characteristics can be obtained all over the domain by means of the calculated wave modes $\phi_n(x), n = -1, 0, 1, 2, 3, \ldots$, using the expansion (4). Also, information concerning distributions of moments and stresses on the plate are obtained from the solution, through the vertical deflection ($w$); see Belibassakis & Athanassoulis (2005).

5. SET UP OF THE SYSTEM AND NUMERICAL RESULTS

The above CMS is solved, using appropriate boundary conditions specifying the wave incidence (see Fig.1) and the boundary conditions Eqs.(3e) and (3f) associated with elastic plate deflection at the edges, enforcing zero shear force and moment. Furthermore, the present model is applied to the solution of the studied problem, in conjunction with the Perfectly Matched Layer (see Colino & Monk 1988). The latter PML permits truncation of the computational domain on the horizontal plane, and its coefficients are optimized as discussed in Belibassakis et al (2001) for absorbing the propagating and scattering waves arriving at the lateral and downwave boundaries of the domain with minimum reflection. The drawback of the PML model is that a layer surrounding the computational domain of width approximately equal to the local wavelength is sacrificed for enforcing the absorbing condition. More details about the present version of the PML can be found in Belibassakis et al (2001) for water waves over 3D variable bathymetric terrains. The same method has been applied very successfully for similar problems in the additional presence of scattering by other sources, as for example in the case of inhomogeneous currents in variable bathymetry regions, as presented in Belibassakis et al (2011).

The discrete version of the present hydroelastic CMS is obtained by truncating the local-mode series (4) to a finite number of terms (modes), and using central, second-order finite differences to approximate the horizontal derivatives. Discrete boundary conditions for both the incident wave and the deflection at the plate edges are obtained by using second-order forward and backward differences to approximate the horizontal derivatives.
Figure 2. (a) Geometrical configuration. (b) Effect of bathymetric variations on the wave field and the elastic plate deflection, in the case of waves of period $T=15\text{s}$ normally incident on a large floating structure over the smooth shoal, as calculated by the present method. The PML region is indicated by using dashed lines.

Figure 3. (a) Effect of the incident wave angle on the modulus of the plate deflection for an elastic plate over a shoal (maximum bottom slope 10%). The three incidence angles examined are: (i) normal incidence, $\theta_i = 0^\circ$ (thin solid line), (ii) oblique incidence, $\theta_i = 30^\circ$ (dashed line), and (iii) very oblique incidence, $\theta_i = 60^\circ$ (thick solid line). (b) Effect of bottom corrugations on the modulus of the elastic plate deflection. The three bottom profiles examined are: (i) horizontal bottom (thin solid line), (ii) undulating bottom with 15% amplitude (dashed line), (iii) undulating bottom with 30% amplitude (thick solid line).

Thus, the discrete scheme obtained is uniformly of second order in the horizontal direction. The coefficient matrix of the discrete system is block structured with 5- and 7-diagonal blocks, corresponding to the discrete versions of the CMS, Eqs.(8) and (9), respectively. The discrete system is numerically solved by means of a parallel implementation making feasible the treatment of realistic domains corresponding to areas with size of the order of many characteristic wavelengths.

In order to illustrate the effects of variable bathymetry (sloping bottom) on the hydroelastic behaviour of the system as modelled by the present method, as a first example we consider in Fig.2 a large elastic floating body ($L=250\text{m}, B=120\text{m}$) modelled as thin elastic plate, with constant characteristics $d=10^5\text{m}^4$, $c=0.005$ and Poisson’s ratio $\nu=0.3$. This floating body is lying over a smooth underwater shoal, characterised by a depth function smoothly varying from $h=15\text{m}$ to $h=5\text{m}$ over a distance of 1.5km, as shown. The mean and maximum slopes are 1% and 10%, respectively. The effect of bathymetric variations on the calculated wave field and the elastic plate deflection, in the case of waves of period $T=15\text{s}$ normally incident on the elastic structure over the smooth shoal, is shown in Fig.2(b). Calculations are based on a grid $151\times121$ grid points on the horizontal domain, and using only the first three modes ($n=0,1,2$) in the present expansion, Eq.(4).

Furthermore, in order to provide an indication concerning the effect of bottom corrugations and of the incident wave angle on the elastic plate deflection, a longer floating body is examined $L=500\text{m}$ in the same environment and conditions as before, which extends considerably in the transverse direction. The flexural of rigidity and the rest parameters of the plate have been kept the same. Results are presented in Fig.3 along the centerline of the floating body. In this case, we observe in Fig.3(a) that as the incident wave angle increases, the horizontal (along the $x$-axis) wavelength of the wave and of the plate deflection increase, as they ought. Also, refraction
phenomena become more significant. On the other hand the effects of bottom corrugations on the hydroelastic behaviour of the system are examined in Fig.3(b) and are found to be of secondary importance.

CONCLUSIONS
In this work, the coupled-mode model developed by Belibassakis & Athanassoulis (2005) is extended and applied to the hydroelastic analysis of three-dimensional large floating bodies of shallow draft lying over variable bathymetry regions. The present formulation finds also useful applications to the study of interaction of water waves with ice floes in coastal waters. A general bathymetry is assumed, characterised by a continuous depth function, joining two regions of constant, but possibly different, depth. Following previous works for the propagation and diffraction of water waves over general bathymetric terrains and in the presence of other inhomogenities, we consider the scattering problem of harmonic incident waves, under the combined effects of variable bathymetry and a large floating elastic structure of orthogonal planform shape. The hydroelastic problem is formulated within the context of linearised water-wave and thin elastic-plate theory. In order to treat the wave field beneath the elastic floating plate, down to the sloping bottom boundary, a complete, local, hydroelastic-mode series expansion of the wave field is used, enhanced by an appropriate sloping-bottom mode. The latter enables the consistent satisfaction of the Neumann bottom-boundary condition on a general topography and accelerates the convergence. First numerical results concerning floating structures over flat and inhomogeneous seabeds are shown, and the effects of wave direction, bottom slope and corrugations on the hydroelastic responses are discussed. Future work is planned towards the detailed investigation of modelling capabilities of the present approach and its extension to treat planform shapes of floating bodies and structures of more general shape.

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