A COUPLED-MODE SYSTEM FOR SHEAR DEFORMABLE BEAMS AND PLATES
OF NON-UNIFORM THICKNESS

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ABSTRACT

Abstract. A new coupled system of horizontal equations is presented, based on the theory of shear deformable plates (or beams), derived by an enhanced representation of the elastic displacement field. The present model contains additional elastic vertical modes, permitting the shear stress to vanish on both the upper and lower boundaries of an elastic body characterized by non-uniform finite thickness. It naturally extends third-order plate theories by Reddy and Bickford (see [1], [2]) to plates and beams of general shape. In the case of beams such a model was introduced and studied by Athanassoulis and Belibassakis [3], in the context of hydroelastic analysis of thick floating ice-sheets. The present coupled-mode system of horizontal differential equations is obtained by means of Kantorovič’s method as applied to the variational principle for the Hamiltonian of an elastic deformable body. The proposed representation of the displacement field extends the models of Reddy and Bickford by introducing additional terms in the expansion, permitting shear stress to vanish at the sloping boundary surfaces of a plate of finite and non-uniform thickness. The extended model is presented and discussed for the case of both beams and plates. Numerical examples concerning modal analysis of beams are presented and compared against FEM solutions. Finally, the utilization of the derived model for the hydroelastic analysis of generally shaped floating bodies of very large horizontal dimensions, such as ice sheets or VLFS lying over general bathymetry regions, is discussed.

1 INTRODUCTION

Plates and beams can be dynamically analyzed by means of the 3D elasticity equations in accordance with their dominating material behavior. In order to avoid special subtleties and pitfalls of 3D numerical methods when applied to thin bodies, beam and plate theories have been developed by assuming specific forms concerning the expansion of the stress or displacement field $u_i(x,y,z;t), i=1,2,3$, with respect to the thickness direction $z\in[-b/2, b/2]$, involving combination of unknown amplitude functions. In the sequel a short review is presented concerning such models for plates (or beams), noting that the methodology can also be used to treat other types of solid bodies having similar geometrical features. Remaining for simplicity to 2D case (corresponding to a beam), where all displacements take place in the $xz-$plane, a generic way to achieve reduction of dimensionality to only the horizontally dependent unknown fields is by means of series expansions of the form

$$u_i(x,z;t) = \sum_{n=0}^{\infty} Z_n^{(i)}(z) u_n(x;t),$$

(1.1)

where $z$ is measured (positive upwards) usually from the mean surface and $Z_n^{(i)}(z)$ are selected to be some basis (or complete system) of functions defined in $z\in[-b/2, b/2]$, where $b$ is the local thickness. The above representation is made efficient by selecting $Z_n^{(i)}(z)$ in such way, leading to fast convergence rates and permitting to restrict the transverse (in the $z$-direction) infinite series to finite sums with small number of significant terms. There are two different lines followed in introducing simplifying assumptions. The traditional approach, initiated by Poisson and Kirchhoff (see, e.g., Timoshenko & Woinowsky-Krieger 1959) gives priority
to physical arguments and states the assumption in terms of physical variables like strains and stresses, by requiring tractions on surfaces parallel to the middle surface are negligible, and strains to vary linearly within the plate thickness. Although the above approach is insightful and very efficient to start with, it has the disadvantage that it becomes very complicated if one tries to proceed to higher-order approximate models and neglects shear deformations. The other approach is mainly based on analytical arguments concerning the form and the nature of expansions (1.1). In this context, on the basis of a Taylor expansion, a very reasonable model is obtained by using \( Z_{\alpha}^{(n)}(z) = z^n \),

\[
 u_i(x, z; t) = \sum_{n=0}^{N^{(i)}} z^n u_{i,n}(x; t),
\]

where the number of terms retained, \( 1 + N^{(i)} \), may be different for each \( i \)-component of the displacement. In all thin plate theories the assumption of transverse (vertical) inextensibility is considered to hold. Thus, in Eq. (1.2) \( N^{(i)} = 0 \), i.e. \( u_i(x, z; t) = w_i(x; t) \). Further, by considering for the rest of displacements the same number of terms \( 1+N \) the various thin plate theories are obtained characterized by means of their order \( N \). For example, first-order theories (e.g., Mindlin 1951) are based on the expansion

\[
 u_i(x, z; t) = u_i(x; t) + z u_i(x; t),
\]

involving, in general, three horizontal surface fields (which in 3D plate bending become five). First-order models include the classical plate theory, which further assumes that the function \( u_i(x; t) \) (and \( u_i \) in 3D plates) are expressed in terms of the deflection \( w_i(x; t) \) through \( u_i(x; t) = -\partial w_i(x; t)/\partial x \). A significant improvement of plate (and beam) theory is provided by third-order models (\( N = 3 \)), involving in general five 2D fields (or nine in 3D plate bending). Among the various third-order models appearing in the literature (reviewed by Wang et al. 2000 and Reddy 2004), we mention here the one developed by Bickford (1982) and Reddy (1984), based on the following displacement expansion:

\[
 u_i(x, z; t) = u_i(x; t) + z u_i(x; t) + \alpha z^2 \left( u_i + \partial w_i / \partial x \right),
\]

where \( \alpha = -4/(3b^2) \); see also Reddy (2004). This representation of the displacement field clearly accommodates for a quadratic variation of transverse shear strains and stresses, vanishing at the top and bottom surfaces of a plate, which is the correct boundary condition violated by all lower (than three) order plate theories. Concerning the satisfaction of the latter condition, the Reddy-Bickford model is consistent only for plates with essentially horizontal boundaries. In the case of more general boundaries, additional terms (or plate modes) are required for the consistent satisfaction of zero shear stress at the boundaries, as also discussed in the case of wave-ice sheet hydroelastic problems by Athanassoulis & Belibassakis (2009). Based on the latter work, in the present paper a coupled-mode system of horizontal equations, which is obtained on the basis of the theory of shear deformable plates (or beams), is studied in the case of vibration problems of of finite thickness beams in vacuo. The model is derived by such an enhanced representation of the elastic displacement field containing additional elastic vertical modes, permitting the shear strain and stress to vanish on both the upper and lower boundaries of the thick floating plate, extending third-order plate theories (see also Wang, Reddy & Lee 2000) to general plates and beams characterized by mildly sloped boundaries. The coupled-mode system of horizontal differential equations is obtained by means of a variational principle composed by the one-field functional of elastodynamics (see, e.g., Graff 1975, Reddy 1984) in the plate region, and applied to cases of hinged and fixed thick beams/plates of general form. Numerical examples concerning boundary value problems and modal analysis of beams are presented and compared against FEM solutions. The present analysis supports optimization of enhanced models for application to complex hydroelastic problems involving large bodies or ice sheets floating over general bottom topography (see, e.g., Belibassakis & Athanassoulis 2005, 2006).

2 FORMULATION

The studied configuration consists of a beam (or plate) of general shape, as e.g., an ice sheet of variable thickness \( b(x) \), and below by a rigid bottom; see Fig. 1. For simplicity we restrict ourselves to a 2D problem, however the method can be straightforward extended to 3D. Also, the upper face of the elongated elastic body is assumed flat-horizontal (e.g. by considering that the static plate deflection to be very small), however, our analysis can be easily extended to the case of a more general upper boundary.
The variational formulation of the studied problem is based on the one-field functional ($\mathcal{F}_1$) associated with Hamilton’s principle in linear elastodynamics (see, e.g., Graff 1975, Reddy 1984). The former functional is defined through $F = U - f_b$ including the difference between the kinetic and elastic energy and the gravity (body) forces $f_b = -\rho_b g \hat{k}$, where $\hat{k}$ denotes the unit vector in the direction of the vertical z-axis. The difference $K - U$ is analysed to: (i) the kinetic energy density $K = \frac{1}{2} \rho \varepsilon \frac{\partial u}{\partial t}$, where $\rho \varepsilon$ is the elastic material density and $u = (u_x, u_y, u_z) = (u_x, u_y, 0, u_z)$ is the displacement vector field in the elastic region $D_e = (-\infty, \infty) \times (b(x) + w, 0)$, and (ii) the elastic energy density is $U = \frac{1}{2} \sigma \varepsilon$, where $\sigma \varepsilon$ denotes the stress and $\varepsilon$ the strain tensor field, which satisfies Cauchy equations $\varepsilon = \frac{1}{2} \left( \nabla u + \nabla u^T \right)$. In the above formulae and in the sequel the standard tensor convention, concerning the summation of repeated indices, is used. Finally, $\mathcal{F}_1$ is obtained by integrating all over the elastic domain and in time,

$$\mathcal{F}_1 = \int_{D_e} \int_{-\infty}^{\infty} dz \, dx \, dt \left( \frac{1}{2} \rho \varepsilon \left( \frac{\partial u}{\partial t} \right)^2 - \frac{1}{2} \sigma \varepsilon - \rho_b g \hat{k} \right) + \int_{-\infty}^{\infty} ds \, dt \left( T_i u_i \right),$$

where the last term in the right-hand side involves boundary forcing specified by the given loads $T_i$. The time-independent component of elastic deformation and stresses are considered to be very small and thus, we concentrate here only on the dynamical part. Otherwise, the effects of static deflection should be included in the definition of the geometrical configuration and the dynamical equations will be finally obtained by subtracting by parts the equations concerning the total (static and dynamic) field from the ones concerning the static field only. Next, taking into account the stress-strain relations in their standard form for isotropic elastic material,

$$\sigma = \lambda \Delta \delta + 2 \mu \varepsilon,$$

where $\Delta = \varepsilon \delta \delta$ is Kronecker’s delta and $\mu = G$, $\lambda = \nu E / (1 - 2\nu)(1 + \nu)$ the Lamé constants, expressed through Young’s modulus ($E$), shear modulus ($G$) and Poisson’s ratio ($\nu$), the functional $\mathcal{F}_1$ is finally expressed in terms of $u_i$. By defining the field $w(x, t) = u_z (x, z = -b(x); t)$, to be the vertical motion of the lower elastic plate boundary, the functional $\mathcal{F}_1 = \mathcal{F}_1[u_i; w]$ becomes a functional on horizontal ($u_i$, $i = 1, 2$) and vertical ($w$) elastic fields. The present variational formulation is completed by calculating the first variation of the above functional and requiring its stationarity, i.e.
where the variation of each term is obtained as the sum of partial variations with respect to the involved fields. Considering the variations to vanish at the ends, the above equation finally results in
\[
\int_0^l \int_{-a}^a dx dz \frac{\partial^2 F}{\partial x^2} \frac{\partial^2 \sigma}{\partial x^2} (x, z) \partial u_x + \int_0^l \int_{-a}^a dx dz \left( \frac{\sigma}{\partial y} \frac{\partial n}{\partial y} (x, z) \right) \partial u_y = 0,
\]
(2.4a)

where \( n_i \) denotes the outward unit normal vector (i.e. \( n_i = \hat{k} \)) on the upper elastic boundary \( z = 0 \), and \( n_i = -\left( b', 0, 1 \right) / \sqrt{b' + b} \) on the lower one \( z = -b(x) \). Here, and in the sequel, a prime denotes differentiation with respect to the argument appearing in the subscript. The differential formulation of the problem is obtained from Eq. (2.4) using the independence of all involved variations. The first term of the left-hand side of Eq. (2.4b) leads to the equations of linear isotropic elasticity in \( D_\varepsilon \). Moreover, the second and third terms by themselves require balance of normal and shear stresses on the elastic plate boundaries. In the sequel we will restrict ourselves to the modal analysis of unloaded systems \( (T_r = 0) \) beams supported at their ends, associated with the solution of corresponding eigenvalue problems. The oscillatory fields are time-harmonic and the determination of the corresponding natural frequencies of the system is part of the solution. In this particular case, Eq. (2.4b) becomes
\[
\int_0^l \int_{-a}^a dx dz \left( \rho \varepsilon^2 \frac{\partial u_x}{\partial t} + \frac{\partial \sigma}{\partial x} \frac{\partial n}{\partial x} \right) \partial u_x - \int_0^l \int_{-a}^a dx dz \left( \frac{\sigma}{\partial y} \frac{\partial n}{\partial y} \right) \partial u_y = 0,
\]
(2.4b)

involving the complex amplitudes of the corresponding fields, i.e. \( u_i(x, z; t) = \text{Re} \left( u_i(x, z) \exp(-i\omega t) \right) \) and similar representations for the rest quantities. In this case on the upper flat horizontal boundary the second term leads to the following two conditions necessitating zero normal and shear stresses
\[
\sigma_{xx} = \sigma_{zz} = 0, \quad \text{on} \ \ z = 0.
\]
(2.5)

Moreover, on the lower elastic plate surface the third term of Eq. (2.4) associated with the variations \( \partial u_y \) and \( \delta w \) of the displacement field leads to
\[
b'_1 \sigma_{xx} + \sigma_{zz} = 0, \quad \text{and} \quad \sigma_{xx} + b'_1 \sigma_{zz} = 0, \quad \text{on} \ \ z = -b(x),
\]
(2.6)

which furthermore ensure vanishing normal and tangential shear stresses on the lower boundary. In the case of forced problems \( (T_r \neq 0) \) where external loads are enforced in the normal direction at the sloping boundary, the satisfaction of zero tangential shear stresses is not trivial and this can be achieved by employing the enhanced representation of the elastic displacement field, described in the next section.

The above variational formulation of the present elastic problem permits us to introduce appropriate specific representations of the unknown elastic displacement and wave potential fields (e.g. consistent vertical expansions of these fields in the elastic and water domain, respectively), and derive novel sets of coupled equations governing the elastic system. The unknown fields in these new equations are the coefficients of the above vertical expansions, which are simply the horizontal mode amplitudes. In this way, a kind of reduction of dimensionality is succeeded, by separating the horizontal space where propagation and other wave phenomena manifest and where also the mode amplitudes are defined, from the cross (vertical) space.

3 ENHANCED EXPANSION OF THE DISPLACEMENT FIELD IN THE CROSS DIRECTION

Taking into account the specific geometric characteristics of the thick plate with horizontal upper and general lower boundary shown in Fig. 1, the present expansion of the complex displacement fields is of the form:
\[
u_i(x, z) = u_0(x) - z n_i(x) + \left( z^2 + \frac{2z^3}{3b} \right) u_i(x), \quad \text{on} \ \ z = -b(x).
\]
(3.1)

where \( u_i(x) \) denotes an additional fourth-order term associated with the satisfaction of zero shear stress on the lower boundary \( z = -b(x) \). This term is needed only in the case of sloping lower plate surface and in the case of a horizontal flat boundary is zero. Based on Eq. (3.1) the non-zero components of the strain tensor are calculated as follows:
\[ \varepsilon_{xx} = \frac{\partial u_x}{\partial x} = u'_x(x) + z w_{xx}(x) + \left( z^2 + \frac{2 z^3}{3 b} \right) u''_{xx}(x) + z^4 u'_{xxx}(x), \quad (3.2) \]

\[ \gamma_{sc} = 2 \varepsilon_{xc} = \frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x} = 2 \left( z + \frac{z^2}{b} \right) u_z(x) + 4 z u'_z(x), \quad (3.3) \]

and thus, \( \gamma_{sc} = 0 \) on \( z=0 \). From Eq. (3.3) we see that if the term \( z^4 u''_x(x) \) is not included in the representation (3.1), then \( \gamma_{sc} = 0 \) on \( z = -b(x) \), which would be correct only in the case of flat surface \( b' = 0 \).

In the case of a forced body with a general boundary \( z = -b(x) \), the plate mode \( u_p(x) \) offers an additional degree of freedom of the present higher-order representation, permitting us to obtain non-zero boundary \( x_z \) shear needed to satisfy the condition of vanishing tangential shear stresses. We conclude this subsection by noticing than in the case of a plate with a second variable upper boundary \( (z = a(x)) \), shown by dashed line in Fig.1) one more additional mode is necessary. In this case, the expansion (3.8) for \( u_z(x, z) \) is amended by an extra fifth-order term \( z^4 u''_x(x) \). This remark can be used as starting point for developing a novel hierarchy of models for plates or beams with non-flat upper and lower boundaries. This extension is left to be thoroughly examined in a future work.

4. THE COUPLED MODE SYSTEM (CMS)

By changing the term \( \left( \partial \sigma_y / \partial x \right) \delta u_y \) appearing in Eq. (2.4c) back to \(- \sigma_y \delta \varepsilon_{xx} \), and use the result, in conjunction with the enhanced representations (3.1), and using the following approximate expression of shear stresses at a mildly sloped lower plate boundary \( (z = -b(x)) \):

\[ b_x \sigma_{xx} + \sigma_{zc} = b_x \frac{E \varepsilon_c}{1 - \nu^2} \varepsilon_{xx} + G \gamma_{zc} = b_x \frac{E \varepsilon_c}{1 - \nu^2} \left( u'_{xx} + b w_{xx} + \frac{b^3}{3} u''_{xx} + \frac{b^4}{6} u'_ {xxx} \right) - 4 G b^3 u_x, \quad (4.1) \]

where the parameter \( \varepsilon_c = 1 + \nu^2 (1 - 2 \nu)^3 \), the present variational principle reduces to the following approximate form:

\[ \int_{z = -b(x)} \int_0^{x(b)} \int dxdzdt \left( - \omega^2 \rho_c (u_z \delta u_x + u_x \delta u_z) + \sigma_{xx} \delta \varepsilon_{xx} + \sigma_{zc} \delta \varepsilon_{zc} \right) - \int dx \left( b_x \delta \sigma_{xx} + \sigma_{zc} \right) \delta u_x = 0. \quad (4.2) \]

In deriving the above equations use has been made of the following relations holding in the present case (see, e.g., Graff 1975, Sec 8.3.1):

\[ \sigma_{xx} = \frac{E}{1 - \nu^2} \varepsilon_{xx} + \nu \sigma_{zc}, \quad \text{and} \quad \sigma_{zc} = \lambda \varepsilon_{zc} = \frac{E \nu}{(1 + 2 \nu)(1 - 2 \nu)} \varepsilon_{xx}. \quad (4.3) \]

We note here that in thin plate theory the effect of \( \sigma_{zc} \) stresses and their integral contribution in the plate section is neglected, which can be recovered by the present model by imposing simplifications. Using the expansions (3.9), (3.10) for \( \varepsilon_{xx} \) and \( \gamma_{zc} \) and the corresponding ones for \( \sigma_{xx} = E \varepsilon_c (1 - \nu^2)^{-1} \varepsilon_{xx} \) and \( \sigma_{zc} = G \gamma_{zc} \), and calculating the expressions of \( \delta u_x = (\delta u_z, \delta u_z) \) in Eq. (3.13) in terms of the newly introduced elastic plate modes \( \delta u_x, \delta u_z, \delta u_{zz} \) and \( \delta \omega \), we obtain from the equation associated with the variation \( \delta w_0 = \delta \omega \):

\[ -\omega^2 \rho_c b w + \omega^2 \rho_x \frac{\partial}{\partial x} \left( \frac{b_x u_{xx}}{2} + \frac{b^3}{6} u''_{xx} + \frac{7 b^4}{6} u'_ {xxx} \right) - M^x_x + (bB)_z = 0, \quad (4.4a) \]

where \( M \) denotes the plate bending moment

\[ M = \int_{z = -b}^{z = 0} z \sigma_{zz} dz = \frac{E \varepsilon_c}{1 - \nu^2} \int_{z = -b}^{z = 0} z \varepsilon_{xc} dz = - \frac{E \varepsilon_c}{1 - \nu^2} \left( b_x w'_{xx} + \frac{b^3}{2} u''_{xx} + \frac{7 b^4}{6} u'_ {xxx} + \frac{b^5}{6} u''_{xxx} \right), \quad (4.4b) \]

and the term \( B \) appearing in the left-hand side of Eq. (4.4a) comes from the contribution of shear stresses on \( z = -b(x) \), and is defined as follows.
Similarly, we obtain the following equations from the terms of the equation (4.3) associated with the variations of plate and beam vibrations in vacuo, and calculating the responses of corresponding forced problems. This is left to be examined in detail in a future work.

5. DERIVATION OF SIMPLIFIED ONE-EQUATION MODEL AND PREDICTIONS AGAINST FEM

By approximately neglecting as small quantities the higher-order horizontal modes and , we obtain from Eq.(4.5) at the leading order . Using the latter result in Eq. (4.4) and considering mildly sloped beam surface , we finally obtain from the previous coupled system the following simplified one-equation model

\[
-\omega^2 \rho_x b \left( u_0 + \frac{b}{2} w_0' + \frac{b^2}{6} u_0'' + \frac{b^3}{9} u_0''' \right) - \omega^2 \rho_x \frac{\partial}{\partial x} \left( b^3 \frac{w_0}{12} \right) - 4Gb u_b = 0 ,
\]

where

\[
M = - \frac{E_e b}{1-v^2} \frac{\partial}{\partial x} \left( b^3 \frac{w_0}{12} \right).
\]

The above equations are finally put in the form

\[
(Dw_0^*)' - \omega^2 \left( mw - \left( mb^2/12 \right) w_0 - \left( mbh/4 \right) w_0' \right) = 0 ,
\]

where \( D = \frac{E_e b^3}{1-v^2} \) denotes the flexural rigidity and \( m = \rho_x b \) is the mass density. We can see from Eq.(5.2) that for elastic bodies of very small thickness \( b/\ell << 1 \), where \( \ell \) denotes length of the beam, the above equation reduces to the classical thin plate/beam model:

\[
(Dw_0^*)' - \omega^2 mw = 0 .
\]

As a first step towards the development of the present coupled method, a specific example based on the above simplified model is presented in Figs. 2-3 concerning free vibration analysis and determination of natural frequencies of hinged and fixed beams of length \( \ell = 50m \), characterised by modulus of elasticity \( E=5GPa \), Poisson's ratio \( \nu = 0.3 \), densities \( 923 \text{ kg/m}^3 \), the values corresponding to ice sheet, Squire et al 1995). In this case, except of uniform beam of finite thickness \( b=1m \), we also consider the effect of inhomogeneous beam in 10m<x<40m, with specific form as shown in Fig.3. In this case the beam thickness varies from \( b=1m \) at the ends of the domain to \( b=3m \) in the middle part. Present simplified model, Eq. (5.2), is solved by using central, second-order finite difference scheme, based on equipartition of the horizontal interval. Also, forward and backward differences are used to model boundary conditions at the beam ends \( x=0 \) and \( x=\ell \).
Figure 2. (a) Homogeneous and (b) inhomogeneous hinged beam. Present predictions using \( N = 61 \) (top), against FEM calculated displacement by ANSYS (bottom figures), using (a) 3200 elements, and (b) 5800 elements. Calculated deflection modes by the present method are normalized with respect to the corresponding \( L_2 \)-norm.

Figure 3. Same as in Fig.2, but for fixed beam.
Table 1: Natural frequencies corresponding to first 3 modes of hinged (Fig.2) and fixed (Fig.3) beams.

<table>
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<th>fixed beam</th>
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<td>(b) ANSYS®</td>
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Present method results obtained by using \( N=61 \) gridpoints are compared in Figs.2 and 3 against FEM solutions obtained by ANSYS®, using 3200 elements in the case of constant thickness beam and 5800 elements in the case of variable thickness beam. Also, results concerning the natural frequencies corresponding to the first 3 modes are listed in table 1, and in the case of homogeneous beams they are also compared against the analytical solution given by \( \omega_n = A_n \sqrt{D/m} \pi^2 \), where or \( A_n = (n\pi)^2 \) (for hinged beam) \( A_1 = 22.4, 61.7, 121.0, \ n=1,2,3, \) (for fixed beam). We conclude that present method provides reasonable predictions, in compatibility with thin plate Beam theory, and in relatively good agreement with FEM solver. Comparisons of the above quantities, in conjunction with the elastic fields, obtained from the solution of coupled system will be presented elsewhere.

CONCLUSIONS

A new coupled system of horizontal equations is presented, based on the theory of shear deformable plates (or beams), derived by an enhanced representation of the elastic displacement field. The present model contains additional elastic vertical modes, permitting the shear stress to vanish on both the upper and lower boundaries of a non-uniform thickness plate. It naturally extends third-order beam and plate theories by Reddy and Bickford (see [1],[2]) to plates and beams of general shape. Numerical examples concerning boundary value problems and modal analysis of beams are presented and compared against FEM solutions. The derived model will further support the development of hydroelastic analysis models of generally shaped shear deformable, floating bodies of very large horizontal dimensions, such as ice sheets or VLFS, lying over general bathymetry regions.

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