ABSTRACT

Standard openings in coastal structures are the flushing culverts at breakwaters, allowing periodic exchange of the harbor basin water leading to improved water quality. These openings involve sudden water depth changes occurring when the incident waves meet these openings and transmitted into the harbour. The wave transformations during wave propagation through flushing culverts are dominated by 3D diffraction effects due to sudden water depth changes, along with the finite width of the culvert. A new coupled-mode model, based on eigenfunctions expansions of the Laplace equation, is developed and applied to the numerical solution of the local 3D wave flow problem at the opening. The harmonic wave field is excited by incident parallel waves. The numerical solution converges rapidly, permitting the series truncation at its first terms. The proposed method fully accounts for the 3D diffraction effects and produces information to couple with mild-slope models describing efficiently wave propagation and transformation in coastal regions.

INTRODUCTION

The problem of wave transformation through gaps and openings in breakwaters has long been of interest of researchers in port engineering. Simple geometries that involve wave diffraction by thin and impermeable semi-infinite breakwaters were early investigated theoretically by [1] and popularized with the diffraction diagrams calculated by [2] and later by [3], who used the method of matched asymptotic expansions proposed by [4,5], with the method of matched eigenfunction expansions. The case of a single-gap breakwater has been studied by [6,4] and of an isolated, finite breakwater segment by [7,8] and more recently, concentrating in random wind-generated waves, [9] investigated both detached breakwaters and breakwater gaps. Similar problems have received considerable attention in acoustics (e.g.[10,11]), elastodynamics [12,13] and electromagnetics (e.g. [14]). A formulation and solution of the problem general to the fields of acoustics, elastodynamics and electromagnetics has been carried out by [15] utilizing a Green's function approach.

Most of the studies that concern port engineering (theoretical, numerical or experimental) concentrate in the investigation of openings that represent the entrance to a harbour or the gaps between series of breakwaters and they have in common that they involve wave diffraction through areas of constant or slowly varying water depths. For example, [16] have used integral equation technique to solve the problem of oblique wave diffraction by a periodic segmented offshore breakwater system. Moreover, the problem of water wave diffraction by a periodic array of line segments representing an offshore breakwater has also been investigated by [5] and [17], utilizing an eigenfunction expansion approach. Also, [18] used a finite-element formulation of the [19] equation to simulate asymmetrical gaps in a thin breakwater which compared well with experiments. On the other hand, [20] have looked into waves propagating through an entrance of finite length. They used two methods to describe the problem of waves propagating from the open sea to a harbour through a channel. The first method used Fourier integrals to describe the open sea and the harbour but eigenfunction expansions to describe the channel. This method was first proposed by [14] for 2D electromagnetic waves and [21] extended the method, so that it
involves propagation of electromagnetic short waves of different wave number corresponding to different water depths between the three distinct regions. Dalrymple et al. [20] applied the method for constant depths, hence the problem reduces into solving the Helmholtz equation, but used reduced finite integrals and effective asymptotic approximations for long waves. They applied a second method to solve the problem, the so-called “buffer” domain method proposed by [22-25] which introduces two additional domains at each end of the channel allowing the use of polar coordinates offshore and in the harbour. Momoi’s buffer method is similar to the one proposed in this paper in terms of describing the problem. Dalrymple et al. [20] combined the above buffer domain method to describe energy absorbing sidewalls in the channel and Hankel expansions to describe the far-fields.

Gaps of narrow width, typically less than one wavelength, are also very often constructed at sites of external harbour structures involving the improvement of water quality within a basin [26-30]. Successful control of water quality is dependent upon periodic exchange of the harbour basin water with the sea water of the open sea [31,32]. This is particularly important for health and environmental quality, especially in warmer climates where biological processes are accelerated. These gaps, often referred to as flushing culverts, are concrete conduits with orthogonal or circular intersection, that work either as open channels with no depth variations between leeward and seaward areas (e.g. [33]) or as large pipes, involving sudden variations in water depth either side of the flushing culvert (e.g. [34]). Underwater placement of culverts is standard internationally; in this case, tidal hydrodynamics is the main mechanism for enhanced flushing. In regions, as the Mediterranean, where the ranges of the tides are low, it is preferred to construct the culverts with their longitudinal axis at sea water level, where wave-induced water circulation is enhanced by the amplification of the velocity field inside the harbour basin. Additionally, this latter position of the flushing culvert, as it is placed away from the seabed, benefits from much reduced sediment transport into the harbour basin. An important criterion that quantifies the satisfactory conservation of the water quality, in a harbour basin with flushing culverts in their external structures, is the water renewal time; i.e. the time needed for the full harbour basin to be replaced with water from the open see. For example, [33] calculated the renewal time for a typical harbour, for the optimal positions of gaps that produce the smallest disturbance in the harbour. Generally, the renewal time is associated with wind- and wave-induced circulation in the harbour basin and therefore, when the tides are insignificant, this is directly associated with the wave transformations during wave propagation through flushing culverts (or in the lee side of the flushing culvert) and, more specifically, with the magnitude of the wave transmission coefficient defined as the ratio of the transmitted wave height to the incident wave height.

The transmission coefficient in such problems has been calculated by [35] where 2-D experimental data are compared with a coupled-mode system of horizontal equations, modelling the evolution of nonlinear water waves in finite depth over a general bottom topography where one of the main problems associated with waves transformation through flushing culverts, the sudden variation in bottom topography, is overcome [36-37]. A physical model which can be characterized as a typical part of a rubble mound breakwater with a flushing culvert and numerical results were presented for waves propagating over regions with submerged breakwaters, simulating flushing culverts that are never perfectly filled with water. These comparisons have shown that the present model provides results in relatively good agreement with the experimental measurements; providing however, some limitations concerning the nonlinearity of the wavefield. However, the 2D approach is inadequate for describing the physics of the phenomenon investigated in this paper. The reason for this is twofold. The 2D approach, firstly, does not allow the problem to be part of a more realistic 3D topography and secondly it excludes the 3D effects due to the finite width of the culverts. These wave transformations have been monitored by the 3D experimental work of [34], showing that the phenomenon is dominated by 3D diffraction effects due to the finite width of the culvert, in conjunction with the sudden changes in water depth. In the case when an opening like a gap or a flushing culvert is located away of the tip of a breakwater, in an area of general but mildly sloped bathymetry, as it usually happens in applications (examples presented and discussed in [34] mentioned above), the full three dimensional problem could be simplified by splitting it into two parts, as illustrated in Fig. 1. The present work focuses on the solution of the local three-dimensional flow problem concerning the wave diffraction and transmission through the opening. Then, the derived solution is exploited to specify suitable boundary conditions in the vicinity of the opening in order to couple this information with solvers of the mild slope equation taking into account general refraction, reflection and diffraction phenomena in the site under consideration and in the presence of coastal structures.

In particular, in the next section, we present the essential elements of a new coupled-mode model, based on appropriate eigenfunctions expansions of the Laplace equation in the
various subdomains, which has been developed and applied to the numerical solution of the local 3D wave flow problem at the opening, as described above. This method has also been used for other non-typical shaped breakwaters (e.g. [38] examined a V-type breakwater). In this study, the harmonic wave field is excited by incident parallel waves. The behaviour of the numerical solution is examined showing that it converges rapidly, and permitting the truncation of the series keeping only its first terms. Subsequently, a simplified model is derived afterwards, by keeping only the propagating mode into the series expansion. This approach can be considered as a 3D counterpart of the plane wave approximation of wave transmission and reflection through abrupt bathymetric variations like steps and troughs (see, e.g., [39-41]). Except of its own interest, the approximate solution provides useful results in the case when the depth discontinuities are small and also enables comparison with results obtained by the eigenfunction expansion method for wave diffraction from breakwater gaps in constant depth, in the case when the thickness of the structure is very small in comparison to the wavelength (see, e.g., [5,17]).

Then, the induced flow due to Stokes drift associated with the propagating cylindrical wave in the region of transmission is calculated, as obtained from the present solution. We consider this flow to be a significant component, along with longshore circulation and other components, concerning the water renewal by physical process and maintenance of water quality in a harbour basin protected by the breakwater. Finally, numerical examples are shown and discussed concerning the application of the present methodology, in conjunction with solvers of the modified mild slope equation, in realistic cases, illustrating the effects of breakwater openings on the wave field in the presence of general depth variations. The present work focused on the description of our method. In order to illustrate its applicability, comparisons of numerical results with experimental data, for various incident wave conditions, are necessary, and this will be subject of future work.

**WAVE FIELD IN THE VICINITY OF THE 3D OPENING**

The geometry of the local wave flow problem is shown in Fig. 1. A breakwater of thickness \( a \), containing a finite opening separates two regions of constant but possibly different depths. The depth at the region of incidence is denoted by \( h^{(o)} \) and in the region of transmission \( h^{(i)} \). The width and the depth of the channel are \( a = 2a \) and \( h^{(o)} \), respectively. The wave field is excited by a parallel wave of angular frequency \( \omega \) which propagates with direction \( \Theta \) \( (180^\circ < \Theta < 360^\circ) \) and is incident to the breakwater. A Cartesian coordinate system is introduced with origin at the mean free surface the z-axis pointing upwards. The three dimensional wave field is represented by \( \Phi(x,y,z,t) = \Re \{ \phi(x,y,z) \exp(-i\omega t) \} \), where \( \phi(x,y,z) \) denotes the complex wave potential.

\[
\begin{align*}
\nabla^2 \phi &= 0, \\
\frac{\partial \phi}{\partial z}
\end{align*}
\]

Where \( g \) is the acceleration of gravity. The above system of equations is supplemented by the appropriate conditions at infinity. The latter impose that far upwave at the region of incidence the wave behaves as an incident, a reflected and a diffracted component, and far downwave, at the region of transmission, as a single outgoing radiated component.

The general representations of the wave potential in the various subdomains are easily obtained by separation of variables. In particular, the wave field in \( d = 1 \) and 5, reads as follows

\[
\phi^{(d)}(x,y,z) = \delta_{d1} \phi_I(x,y,z) + \sum_{m=0}^{n} A_m^{(d)} \cos(m\theta) \frac{H_1^{(2)}(k_m^{(d)} \rho)}{H_0^{(2)}(k_m^{(d)} a)}
\]

where

\[
\rho = \sqrt{x^2 + (y - 0.5 \text{sign}(y) a)^2}, \quad \theta = \tan^{-1}\left(\frac{y - 0.5 \text{sign}(y) a}{x}\right)
\]

In the above equation \( \delta_{d1} \) denotes Kronecker’s delta and \( \phi_I \) is a harmonic wave component corresponding to the incident and reflected wave at the region of incidence.
\[ \varphi_d(x,y,z) = \left( \exp\left(ik_r r \cos(\theta-\theta_s)\right) + \exp\left(ik_r r \cos(\theta+\theta_s)\right) \right) Z_n^{(l)}(z) \]

with \( Z_n^{(l)}(z) = \cosh\left(k_d (z+h)\right)/\cosh(k_d h) \),

where \( k_d = k_n^{(l)} \) is the wavenumber of the propagating mode at the region of incidence, given by the positive real root of the dispersion relation at the depth \( h \) (see below Eq. (5a), for \( d=1 \)). The corresponding representations of the wave field in the other subdomains are for \( d = 2 \) and 4,

\[ \varphi_d(x,y,z) = \sum_{n=0}^{\infty} \left[ A_{nn}^{(d)} \cos(n\theta) J_{n+1}(k_{n+1}^{(d)} \rho) + B_{nn}^{(d)} \sin((n+1)\theta) J_{n+1}(k_{n+1}^{(d)} \rho) \right] \]

and for \( d = 3 \),

\[ \varphi_d(x,y,z) = \sum_{n=0}^{\infty} \left[ \cos(\kappa_n(x+a)) \left[ A_{nn}^{(d)} e^{i\kappa_n y} + B_{nn}^{(d)} e^{-i\kappa_n y} \right] \right] \]

where \( \{\kappa_n, n=0,1,2,...\} \) and \( \{\lambda_m, m,n=0,1,2,...\} \) are given by

\[ \kappa_n = n\pi/a_n = n\pi/2a \quad \text{and} \quad \lambda_m = \sqrt{\left(k_m^{(d)}\right)^2 - \left(k_n\right)^2} . \]

In the above equations the functions \( \{Z_n^{(d)}(z), m=0,1,2,...\} \) and the numbers \( \{k_n^{(d)}, m=0,1,2,...\}, \) \( d = 1,5 \), are eigenfunctions and eigenvalues of vertical Strum-Liouville problems formulated in the intervals \( 0 < z < h_2 \) respectively. The eigenvalues \( k_n^{(d)}, i k_n^{(d)}, i k_2^{(d)}, \ldots \), in each subdomain are obtained as the roots of the dispersion relation

\[ \omega^2 = kg \tan(h h_2), \quad d = 1,\ldots,5, \]

and the corresponding eigenfunctions are defined by

\[ Z_n^{(d)}(z) = \cosh(k_n^{(d)} \left(z+h_2\right))/\cosh(k_n^{(d)} h_2) . \]

The scaling of Hankel and Bessel functions in Eqs. (2a) and (3) and of the exponential functions \( e^{i\lambda_m} \) in Eq. (4a) has been introduced in order to avoid overflow problems for the evanescent modes due to rapidly growing and decaying exponential behaviour of these terms (see also, [42]). We remark here that when the arguments of the previous functions become imaginary, as it happens to be the case of evanescent modes \( (m \geq 1) \), the corresponding functions turn to appropriate forms involving modified Bessel functions

\[ H_n^{(1)}(ik_n^{(d)} \rho) = -\frac{2i}{\pi} \exp\left(-\frac{n\pi i}{2}\right) K_n(ik_n^{(d)} \rho) \]

and

\[ J_n(ik_n^{(d)} \rho) = \exp\left(\frac{n\pi i}{2}\right) f_n(ik_n^{(d)} \rho) \]

ensuring the completeness and consistency of representations (2a) and (3).

**FIGURE 3.** HORIZONTAL PLOT OF THE CALCULATED WAVE FIELD IN THE VICINITY OF THE OPENING IN THE CASE OF (a) NORMALLY AND (b) 45deg-OBLIQUELY INCIDENT WAVES OF PERIOD \( T = 6 \)sec ON THE OPENING OF MAIN DIMENSIONS \( a_x = 10m \) AND \( a_y = 15m \).

**FIGURE 4.** VERTICAL PLOT OF THE SOLUTION IN THE CASE OF NORMALLY INCIDENT WAVES OF FIG 3 \( (h(0)=5m, h(0)=4m \) AND \( h(0)=1.5m ) \).
The complete solution of the 3D problem of wave flow around the opening is then obtained by calculating the coefficients $A_{m,n}^{(d)}$, $d=1,5$ and $B_{m,n}^{(d)}$, $d=2,3,4$ involved in the above expansions in order to satisfy the matching conditions concerning continuity of the wave potential and its normal derivative at all points on the vertical interfaces $I_{0}^{(d)}$, $d=1,4$, separating the various subdomains, following Galerkin approach based on the completeness of the systems of vertical eigenfunctions, and leading to a set of coupled projection-type equations. The linear coupled-mode system with respect to the unknown coefficients $A_{m,n}^{(d)}$, $d=1,5$ and $B_{m,n}^{(d)}$, $d=2,3,4$ is then obtained by substituting the representations for $\phi^{(d)}$ from Eqs.(2,3,4) in the above equations and performing the integrations (analytically). The discrete version of the system is constructed by truncating the series keeping a finite number of terms $M$ and $N$ in the corresponding summations and considering only the first equations for $k=0, M$, and $\ell$.

Thus, the total number of unknowns which equals the dimension of the discrete system is $N_{d} = 8(M+1)(N+1)$. The forcing of the coupled mode system involves data concerning the incident wave and its derivative on the vertical interface $I_{12}$. Details concerning the structure of the linear algebraic system including the expressions of the matrix-coefficients and the right-hand and the convergence characteristics of the method will be presented elsewhere.

As an example the calculated wave field in the vicinity of the 3D opening of period $T=6$sec on a breakwater containing an opening of width $a_{o}=10m$ and channel length $a_{c}=15m$ is shown in Fig. 3, in the case of normally and (b) obliquely incident waves. The depths around the opening are $h^{(o)}=5m$, $h^{(i)}=4m$ and in the channel $h^{(o)}=1.5m$, as shown in the transect along the centerline ($x=0$) of the channel plotted in Fig. 4, where also the details of the wave field are illustrated by using equipotential lines. Also, in the same figure the values of the wave potential at the free surface are plotted by using a thick line, which is proportional to the free surface elevation. We clearly observe in the previous figures the strong modification of the wave field in all subdomains and that the equipotential lines intersect the bottom-profile both horizontal and vertical wall parts perpendicularly which is evidence of the consistent satisfaction of the no-entrance bottom boundary condition.

**Simplified Model Based on Helmholtz Equation**

A simplified model of the local 3D problem in the vicinity of the opening is derived by keeping only the propagating mode ($m=0$) in the expansions (Eqs 2,3,4) of previous section. In this case the dependence of the solution from the vertical coordinate separates from the horizontal ones, for all subdomains $d=1,2,..,5$, is approximated by

$$\phi^{(d)}(x,y,z) = Z_{0}^{(d)}(z)\phi^{(d)}(x,y),$$

where functions $Z_{0}^{(d)}(z)$ are given by Eq.(5b) for $m=0$, and $\phi^{(d)}(x,y)$ are defined by the inner summations (with respect to index $n$) in the right-hand side of Eqs.(2), (3) and (4). It is clearly seen that in this case the complex amplitude $\phi^{(d)}(x,y)$ associated with the scattering field satisfies the homogeneous Helmholtz equation on the horizontal plane ($z=0$),

$$\nabla^{2}\phi(x,y) + k^{2}\phi(x,y) = 0,$$

where $\nabla_{n} = (\partial/\partial x, \partial/\partial y)$, which is governed by three distinct, but constant wavenumbers, in the region of incidence, in the region of transmission, and in the channel, respectively, obtained as the positive real roots of the dispersion relation, Eq. (5a), formulated at the corresponding depths $h^{(o)}$, $h^{(i)}$ and $h^{(c)}$. The solution in the present case is obtained by enforcing appropriate continuity conditions on $\phi^{(d)}(x,y)$ on $z=0$. In this case, a modification of the projection technique described in the previous section is applied to derive the numerical solution, which is based on the completeness of the systems

$$\{Z_{1}^{(d)}(z)\cos(\ell\cdot), \text{ for } d=1,2 \text{ and } d=4,5, \text{ in } (0, \pi) \text{ and (}x,2\pi\), respectively, as well as of the system

$$\{z_{1}^{(d)}(z)\cos(\kappa_{l} f + a)\} \text{ in } (-a,a).$$

Being an approximation of the 3D problem, the simplified model provides useful results in the case of small depth variations from the region of incidence towards the region of transmission. Except of its own interest as an analytical solution of the Helmholtz equation governed by three distinct and possibly different wavenumbers, it also enables comparisons with eigenfunction expansion techniques treating wave diffraction from breakwater gaps in constant depth, in the case when the thickness of the structure is very small in comparison to the wavelength developed by [5], and extended to general wave incidence by [17]. A result concerning the calculated diffraction parameter by the present simplified method based on Helmholtz equation on the horizontal plane, in the case of single gap breakwater of limiting thickness is shown in Fig. 5, for normally incident waves ($\theta = 270^\circ$), and $15\text{deg}$-obliquely incident waves with respect to the centerline of the opening ($\theta = 255^\circ$) and $ka=\pi$. To simulate the small thickness we have used $a_{o}=1m$, $a_{c}=5m$ and constant depth $h=4m$. We observe in Fig. 5 that, as the breakwater thickness becomes smaller, the calculated patterns by present simplified method become compatible with the ones presented by [17] for zero thickness ($a_{c}=0$).

**Induced Flow in the Transmission Region**

The far-field asymptotics in the region of transmission are easily obtained from the present coupled-mode solution by ignoring the effects of evanescent modes ($m>0$) that are rapidly (exponentially) decaying away from the opening and
transmission of constant depth $h_i$. In the above equation, $\rho = \sqrt{x^2 + (y + a_y)^2}$, $\theta = \tan^{-1}\left((y + a_y)/x\right)$. By denoting the corresponding Kochin function as follows

$$f(\theta) = \sum_{n=0} \hat{A}_n \cos(n\theta) \exp\left\{i\left(kh - \frac{n\pi}{2} + \frac{\pi}{4}\right)\right\},$$

(11)

the wave amplitude in the far downwave region of transmission, normalized with respect to the incident wave amplitude $H/2$ (where $H$ is the incident wave height), is then given by

$$\frac{A^{(i)}}{H/2} = \frac{\left|\phi^{(i)}(x, y, z = 0)\right|}{H/2} \approx \frac{2}{\pi k^{(i)} \rho} \left|f(\theta)\right|.$$  

(12)

Furthermore, in large horizontal distances in the region of transmission the wave crests become flattened. Thus, the volumetric flux associated with Stokes drift per unit length of wave front of the transmitted cylindrical wave is easily obtained as [42] as follows

$$\frac{U}{\delta s} = \frac{H^2}{8} \frac{k^{(i)} c}{\tanh\left(k^{(i)} h_i\right)},$$

(13)

where $c = \sqrt{g \tanh\left(k^{(i)} h_i\right)/k^{(i)}}$ denotes the phase speed in the region of transmission and $\delta s$ the differential length along the wave crest. Using the above equations and integrating along semicircular arc we finally obtain the following estimation of the volumetric flux

$$\frac{Q}{cH^2} = \frac{1}{4\pi \tanh\left(k^{(i)} h_i\right)} \int_{\theta = -\pi}^{\pi} \left|f(\theta)\right| d\theta.$$  

(14)

Furthermore, if we neglect all coefficients $A_n$, $n \geq 1$, keeping only the first term $A_0$ in the expansion we obtain the following approximation

$$q = \frac{Q}{cH^2} \approx \frac{\left|\phi_0\right|}{4 \tanh(k h_i)}.$$  

(15)

The above results could be used in conjunction with the simplified model based on the propagating mode to obtain an approximation of the wave induced flow. This result could be useful in calculating the various flow components permitting the estimation of the water renewal process in the protected area or basin by the breakwater.

As an example, we present in Fig. 6 the obtained prediction concerning the flux parameter ($q$) against the non-dimensional wavenumber ($kh$), in the case of the gap of the thin-wall breakwater of Fig. 5(a), considered in constant depth $h = h^{(i)} = h^{(0)}$ and normal wave incidence ($\theta_i = 270^\circ$).

We observe in this figure that the calculated flow rate could achieve significant values, especially for intermediate and shallow wave conditions, which could be found useful in the design flushing culverts at breakwaters aiming to an
improvement of the water quality.

NUMERICAL RESULTS AND DISCUSSION
As a standard model for describing wave propagation, including refraction and diffraction effects, in coastal regions, characterized by variable bathymetry \( h(x,y) \) and in the presence of structures, we consider the modified mild-slope equation [44-45],

\[
\nabla_{\phi}^2 \phi(x,y) + \frac{\nabla_{\phi} (c_{\phi})}{c_{\phi}} \nabla_{\phi} \phi(x,y) + k^2 (1 + \psi) \phi(x,y) = 0 , \quad (16)
\]

where \( c = c(x,y) \) and \( c_{\phi} = c_{\phi}(x,y) \) are the local phase and group velocities, respectively, and \( k(x,y) \) is the local wavenumber associated with the propagating mode. The solution of the above equation provides the spatially varying complex wave amplitude over the examined area. Subsequently, the 3D wave field is obtained as follows

\[
\phi(x,y,z) = \phi(x,y) \frac{\cosh(k(z+h))}{\cosh(kh)} . \quad (17)
\]

The function \( \psi = \psi(x,y) \) appearing in Eq. (16) is dependent on the gradient and the curvature of the depth function (see, e.g., [46-47]). The above model, in conjunction with the PML absorbing model (see [48]), will be used in this section to derive results in coastal areas in the presence of breakwater, and to illustrate the effects of openings on the calculated wave field. Following the above approach, in the present work the coefficients of the second-order (horizontal Laplacian) operator in Eq. (17) are modified within an absorbing layer of thickness.
FIGURE 9. DIFFRACTION PARAMETER AS CALCULATED WITH THE PRESENT METHOD FOR 45deg-OBLIQUELY INCIDENT WAVE OF PERIOD $T=5s$, IN WATER OF CONSTANT DEPTH $h=4m$: (a) WITHOUT AND (b) WITH THE EFFECT OF THE OPENING IN THE BREAKWATER.

of the order of the characteristic wavelength $\lambda = 2\pi / k$ around the west ($x = x_1$) and southern ($y = y_1$) sides of the domain, as shown in Fig. 7. Details about the implementation of the PML model for water-wave problems in variable bathymetry regions are given in [49]. On the other hand at the north ($y = y_2$) and eastern ($x = x_2$) sides of the computational domain appropriate boundary conditions are used specifying the incident and reflected wave.

A first example illustrating the effect of an opening of width 80m in a straight and long breakwater of thickness 10m is shown in Fig. 8, for normally incident waves of period $T=15s$. Results presented are normalized with respect to incident wave amplitude. The water depth is the area is $h=10m$. In this case a 121x121 grid is used. We clearly observe an absorbing effect of the PML at the west and south sides of the computational domain. Also, the diffraction effect due to the tip of the breakwater and the operation of the opening are clearly resolved in Fig. 8. Another example concerning the diffraction parameter as calculated present method is shown in Fig. 9. In this case we consider a 45deg-oblique incident wave of period $T=5s$, in water of constant depth $h=4m$. Again, the effect of the opening is quite strong, modifying the transmission coefficient at the lee side of the breakwater immediately after the location of the opening from 20% to about 45%.

CONCLUSIONS

A numerical coupled-mode model is developed and applied to the solution of the local 3D wave flow problem in the vicinity of openings of coastal structures. Such types of openings are used as flushing culverts in breakwaters to allow the periodic exchange of the harbour basin water leading to an improvement of the water quality. The openings involve sudden changes in water depth and channelling of incident harmonic waves that are transmitted into the lee side of the coastal structure. The present method is based domain decomposition, in conjunction with appropriate eigenfunctions expansions of the Laplace equation in the various subdomains. The behaviour of the numerical solution is examined showing that it converges rapidly, permitting the truncation of the series keeping only its first terms. The proposed method fully accounts for the 3D diffraction effects and produces useful data and information to couple with mild-slope models treating the general wave propagation and transformation in the coastal region, in the presence of breakwaters with gaps and 3D openings. To illustrate the accuracy and applicability of the present method, numerical results will be compared with extended experimental data for various wave conditions, and this will be subject of future work.

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